

EXTERNALITIES AND ASYMMETRIC INFORMATION*

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A reconsideration of the Pigovian theory of regulating externalities via taxation is undertaken for environments with private information. The presence of private information may have no effect on the social optimum; but when it has an impact, it is to cause a group of different agents to share the same production or consumption levels. The model developed provides an appealing characterization of when such situations transpire: they occur when the individuals who desire most to engage in some activity are the ones who society least wants to participate. Since such instances could potentially be regulated by the imposition of quantity controls, this may explain authorities' apparent predilection for quantity limits rather than tax-cum-subsidy schemes to manage many externalities.

I. INTRODUCTION

In the textbook model of externalities, the size of the external effects depends solely on the quantity of the commodity produced and not upon who produces it [Pigou, 1960; Meade, 1952]. For example, it is irrelevant whether two firms produce 50 tons of noxious pollutants each, or one discharges 100 tons of effluent and the other zero. In such a world the external effects of pollution can be efficiently regulated by imposing a per unit effluent tax that is the same for all producers.

Some externalities, however, do not share this irrelevance of the identity of the producer, and this is the subject of the current paper. In particular, consider the case of education. The external effects of education have been widely discussed with one commonly identified source being the generation of ideas and technology; education is an input to research and development. The external effect lies in the inability of inventors to completely capture the benefits of R&D.¹ Even a monopolist fails to capture fully the entire surplus created from a product if he cannot price discriminate completely. Thus, one external effect in education arises from

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1. In particular, some intellectual achievements, e.g., mathematical theorems, cannot be patented and are inputs into production (theorems are an input into engineering). Copycat inventions and finite patent life also insure that the benefits of invention do not generally accrue solely to the inventor.

public goods created by the educated. Lucas [1988] and Romer [1986] have emphasized the importance of external effects associated with knowledge as a contributor to economic growth. Education is also commonly thought of as yielding external benefits by strengthening the social fabric through fostering notions of mutual respect and cooperation among individuals, ingredients most people view as necessary for a civilized society. Likewise, education may be beneficial in persuading society's citizenry to follow certain practices essential for reasons of public health and safety. Whatever the precise benefits from education are, Azariadis and Drazen [1990] and Barro [1991] present evidence suggesting that countries' growth rates are positively correlated with past investment in human capital.

The external benefits from education may depend on the *type* of person who is being educated. It may be desirable for society to instill a certain set of minimal traits in its citizenry. How easily or willingly an individual will adopt these traits may depend upon certain personal attributes, such as his or her ability to learn.² For instance, intelligent people may be able to deduce from first principles that storing gasoline next to a furnace is ill-advised, while less-intelligent people may need to be taught this.³ Since one has an interest in the fire safety of his neighbors, he values their education and *values it more highly the less intelligent they are*. Similarly, society tends to benefit from advances in arts and sciences brought about through creative genius. The marginal social value of providing a unit of education to a genius may be higher than giving it to a person of median intelligence. Nobody would suggest giving a course in quantum mechanics to somebody who finds calculus impenetrable. The above two examples suggest that it may be reasonable to speculate from the *social* perspective that the provision of education should be a U-shaped function of type: other things equal, a unit of education is worth more at the

2. Given the current state of the social sciences, this discussion necessarily is a bit loose. It is doubtful whether ability to learn (type) can be ordered on a one-dimensional scale, but suppose for the following discussion that it can be anyway. Also, whether ability to learn is primarily genetically or environmentally determined (a controversial issue in psychology) is irrelevant for the analysis to be undertaken. All that matters here is that people differ in their ability to learn, for whatever reason, and that this ability can be ordered on a simple scale.

3. The word intelligence is being used loosely. For purposes of discussion, let it simply denote an individual's ability to learn.

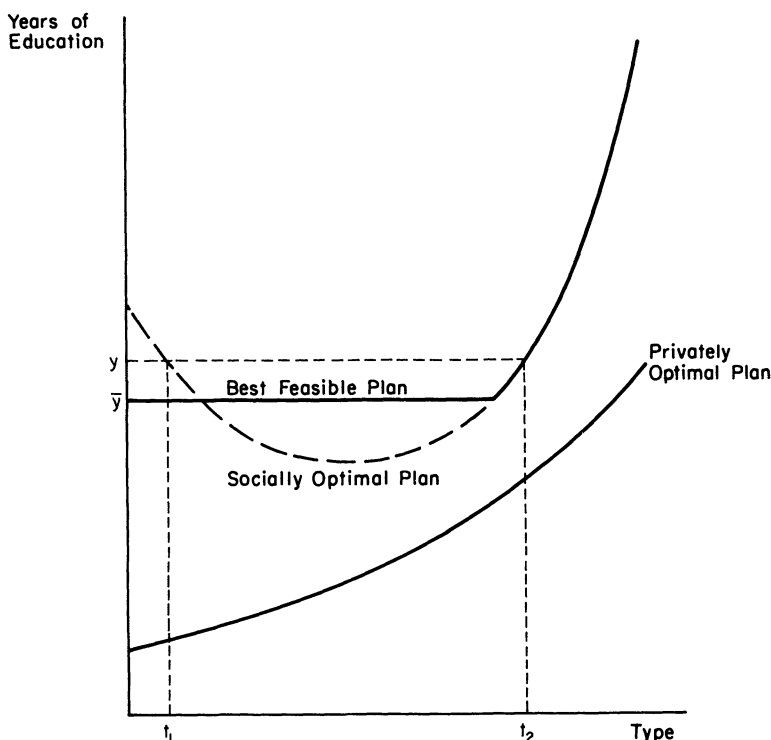


FIGURE I
Optimal Plans Given a Fixed Unit Cost of Education

low and high ends of the scale (see Figure I). Note, in support of this proposition, that grade schools expend proportionately more resources on slow learners and on gifted children than on the median student.

Insofar as type is observable, a socially optimal education program could be implemented with type-dependent subsidies. If subsidies cannot be conditioned upon type, however, because type is unobservable, then the socially optimal education plan may not be feasible. To see why, suppose that the *private* demand for education is increasing in type. Then higher types are willing to pay more for a given amount of education than lower types. Now, in order to persuade agents at the lower end of the spectrum to acquire the prescribed amount of education, they must be given a disproportionately high subsidy relative to other agents on the

scale. But at this high rate of subsidy, other agents have an incentive to pretend that they are low types so as to obtain more education at a lower price than is socially desirable for them.⁴ For instance, a subsidy designed to entice an agent of type t_1 to obtain y units of education would prove to be even more attractive to agent t_2 , who under the proposed plan should get the same amount of education but at a higher price. In such a circumstance, the best feasible plan could be to offer the same amount of education \bar{y} at a constant price to everybody at the lower end of the spectrum. Note that at the other end of the scale this problem will not manifest itself. The minimal subsidy required to induce high type agents to take the requisite amounts of education will not be enough to attract low demand agents to the left of them on the spectrum. This theory is in accord with educational practices in most Western countries. Minimum levels are imposed on everybody, with subsidies being provided at higher educational levels. Despite the theory's abstraction from many important real world considerations, it seems to accord reasonably well with the facts.

There seem to be many externalities that share, at least to some extent, the joint features that the size of the externality depends on the type of agent consuming (or producing) the good in question, and that type is private information. A formal investigation of such situations will be undertaken here. It will be shown that while the existence of private information may have no impact on the optimal regulation of the externalities, when it has an effect it will cause a group of heterogeneous agents to share the same consumption level. That is, for a subset of agent types, quantity consumed is constant, where it would not be but for the private information. In extreme cases quantity may be constant across all agents. Whether or not the presence of private information constrains the social optimum, the welfare-maximizing allocation can be supported by a *nonlinear* price system. Those situations that call for constant quantities across individuals, however, could also

4. As a general proposition, the regulator may be able to determine (though imperfectly) an agent's type through testing and other schemes. Imagine testing agents here for their intelligence level. Note that such testing is unlikely to be effective, though, for verifying declared type at the lower end of the scale. This occurs since it would always be possible for high type agents to conceal their true ability by performing poorly on tests, when it was in their interest to do so. Testing is more likely to be accurate at the upper end of the scale with, for instance, universities frequently conditioning entrance on test scores.

be implemented with simple quantity limits.⁵ The formal theory developed is silent on which method of implementation will be chosen; they both achieve the *same* social optimum. It may be the case, though, that the administrative burden of regulating via quantity limits is lower. To this limited extent, the current analysis may provide an explanation for government's apparent preference for quantity limits over taxation for regulation.

In the next section a model of a world with externalities and private information will be presented and analyzed. In any regulated market where activity is not directly observable, incentives may emerge for private agents to try to thwart the regulator's original plan. Efficient regulation necessitates that the regulator incorporates such possibilities into the initial design of his scheme. In the third section, the effects of black markets, where some agents can cheat at a cost on the legally prescribed allocations, are considered. Finally, concluding comments are offered in the last section.

II. THE MODEL

Consider a closed economy inhabited by a continuum of agents. Agents are randomly distributed by type, denoted by t , over the interval $[0, 1]$ according to the continuous density function $f(t)$. An individual's type is private information. Agent t 's goal in life is to maximize his utility $\underline{U}(t)$, as given by the function,

$$(1) \quad \underline{U}(t) = U(x(t), y(t), t) + \int_0^1 \epsilon(y(s), s, t) f(s) ds.$$

The first term is the direct utility agent t obtains from his personal consumption, $x(t)$ and $y(t)$, of two goods X and Y . The direct utility function $U(\cdot)$ is assumed to be increasing, concave in its first two arguments, and twice continuously differentiable. The second term is the indirect utility—which may be negative—the agent realizes

5. Weitzman [1974] provides a precursor to this work. Weitzman discusses the relative benefits of regulating economic activity by price versus quantity limits. In his analysis the cost and benefits of producing some good are of uncertain magnitude. It is assumed that the regulator must pick the price or quantity of production before the resolution of this uncertainty. The conditions under which the control of the production activity is better on average via price or quantity regulation are examined. This turns out to depend upon the relative curvature of the marginal benefit and cost schedules. A different set of issues from these is being addressed in the current study: optimality over all mechanisms for regulating the externality.

from the consumption of Y by other agents in the economy. That is, there are externalities in consumption present. More specifically, the term $\epsilon(y(s), s, t)$ measures the utility benefit to individual t from agent s 's consumption of $y(s)$ units of Y . Also, suppose that

$$(2) \quad U_{yt}(\cdot)/U_y(\cdot) > U_{xt}(\cdot)/U_x(\cdot) \quad \text{for } t \in [0, 1].$$

This restriction implies that an agent's marginal rate of substitution for good Y is an increasing function of his type. Since demand is characterized by the efficiency condition $U_y/U_x = p'$, where p' represents the relative price of Y , this condition may be also viewed as requiring higher types to have unambiguously higher demand for Y at every price.⁶

Each individual t is endowed with a certain amount of the X good, \bar{x} . It is assumed that commodity X can be transformed into Y by agent t according to the following linear production technology:

$$(3) \quad y = x/c.$$

Before agents have been randomly assigned their type, it is in their interest to coalesce and mutually agree on a system to govern society's future consumption allocations. Recall that an agent's type is private information, so that any allocation mechanism conditioning on an individual's true type must ensure, if it is not to be thwarted, that agents will in their own self-interest end up truthfully revealing their type. That is, the allocation mechanism must be incentive compatible. The notion of incentive compatibility will now be characterized.

Without any loss of generality it may be presumed that an allocation mechanism is incentive compatible; see Harris and Townsend [1985] and Myerson [1982]. This implies that under an allocation system the utility an agent earns from revealing his type t honestly must be at least as great as that which would be realized if the individual claimed he was some other type s instead. In particular, to achieve an allocation $(x(t), y(t))$, a mechanism must ensure that the following incentive-compatibility condition is obeyed.

$$(4) \quad U(x(t), y(t), t) \geq U(x(s), y(s), t) \quad \text{for } s, t \in [0, 1].$$

Rather than work with the above condition directly, it turns out to

6. This restriction on preferences is often referred to as the "single-crossing" property, and implies that the indifference curves for different types of agent can only intersect once. Cooper [1984] provides a discussion of the importance of the single-crossing property for self-selection models.

be easier to use two equivalent conditions, which are given in the theorem below.

THEOREM [Guesnerie and Laffont, 1984]. The incentive-compatibility condition (4) is *equivalent* to the following two conditions, (5) and (6), holding simultaneously:

$$(5) \quad \frac{d}{ds} U(x(s), y(s), t) \big|_{s=t} = 0$$

and

$$(6) \quad y(t) \text{ is nondecreasing.}^7$$

Thus, the set of incentive-compatible mechanisms ensures that agents placing a relatively high value on commodity Y (high t 's) actually receive more Y , but less X , than those individuals valuing X relatively more, a fact evident from (5) and (6).⁸ Clearly, a feasible mechanism cannot give more of both goods to any particular type of agent. All individuals would claim to be this type. Similarly, a mechanism that provided Y -loving agents with relatively large amounts of X would also not be feasible. These agents (high t 's) would have to be given disproportionately large amounts of X to entice them to reveal their type honestly. But then the X -loving agents (low t 's) would claim to be Y -loving ones. Finally, note that the set of *feasible* incentive-compatible mechanisms appears to be quite large.⁹

7. x, y need not be differentiable, and in this case (5) is replaced by the pseudoderivative conditions,

$$\lim_{\substack{r \rightarrow t \\ r > t}} \sup_{t \leq s \leq r} (u(x(s), y(s), t) - u(x(t), y(t), t)) / (s - t) \leq 0$$

$$\lim_{\substack{r \rightarrow t \\ r < t}} \sup_{r \leq s \leq t} (u(x(s), y(s), t) - u(x(t), y(t), t)) / (s - t) \geq 0.$$

The monotonicity of x and y , along with the differentiability of u , make applications straightforward. See Guesnerie and Laffont [1984] for a complete analysis.

8. Spence [1980] derives a similar result in a model where there are a finite number of types of agents who have preferences which are additively separable in the two goods. Cooper [1984] proves the necessity part of the above theorem, without establishing sufficiency.

9. It is easy to allow an agent's endowment of X to be an increasing function of t . In this case assume, in addition to (2), that the following restriction on preferences is satisfied: $(2') -U_{xx}(\cdot)/U_x(\cdot) \geq U_{yx}(\cdot)/U_y(\cdot)$ for $t \in [0, 1]$; i.e., Y is a normal good. Note, a fortiori, that the demand for Y is increasing in type. Once again, (5) and (6) turn out to be necessary and sufficient conditions for (4) to hold. The formal proof of this statement is a straightforward extension of the theorem presented in Guesnerie and Laffont [1984]. The rest of the analysis of the current paper carries through for this case with little modification.

A characterization of the optimal mechanism governing allocations in society when there are externalities in consumption will now be provided. To make the analysis more tractable, a simplifying assumption is imposed: it will be assumed that agents' utility functions are separable and linear in x so that $U(x(t), y(t), t) \equiv x(t) + V(y(t), t)$. The optimal mechanism is designed to maximize ex ante social welfare W . It is described by the solution to the following programming problem that maximizes the expected value of an agent's utility function (7), subject to society's production possibilities as described by (8), and the incentive-compatibility constraints (5') and (6):

$$(7) \quad \max_{x(t), y(t)} W = \int_0^1 [x(t) + V(y(t), t) + \int_0^1 \epsilon(y(s), s, t) f(s) ds] f(t) dt,$$

subject to (6) and

$$(8) \quad \int_0^1 x(t) f(t) dt = \bar{x} - c \int_0^1 y(t) f(t) dt$$

$$(5') \quad x'(t) + V_y(y(t), t) y'(t) = 0.$$

This programming problem can be rewritten in a simpler form by substituting the constraint (8) into objective function (7) and reversing the order of integration on the $\epsilon(\cdot)$ term to obtain

$$(9) \quad \max_{y(t)} W = \int_0^1 [\bar{x} + V(y(t), t) - cy(t) + \int_0^1 \epsilon(y(t), t, s) f(s) ds] \\ \times f(t) dt \equiv \int_0^1 S(y(t), t) f(t) dt,$$

subject to (6), where the incentive-compatibility condition (5') can be eliminated since $x(t)$ no longer enters the objective function, and hence given the optimal $y(t)$ path, $x(t)$ can be innocuously chosen to satisfy this condition. The term $S(y(t), t)$ represents individual t 's contribution to social welfare W from his consumption of the goods X and Y , both directly via his own utility level and indirectly through the utility level of others. The properties of $S(y(t), t)$ play a crucial role in the design of society's allocation system. For simplicity, presume that S is concave in y . Note that if ϵ is concave in y , then S is as well.

Consider the benchmark case where the incentive-compatibility constraint (6) does not have to be incorporated into society's allocation mechanism, such as would occur in the situation where all information were public. Then the "first-best" social optimum for the consumption of Y for agent t , denoted by $y^*(t)$, would be

described implicitly by the solution to¹⁰

$$(10) \quad S_y(y^*(t), t) = [V_y(y^*(t), t) + \int_0^1 \epsilon_y(y^*(t), t, s) f(s) ds] - c = 0$$

for $t \in [0, 1]$.

Clearly, equation (10) sets the marginal social benefit of agent t 's consumption of Y , represented by the term in brackets, equal to its marginal social cost in terms of forgone X or c . Now in the environment being modeled where information is private, an incentive-compatible mechanism may not be able to obtain this ideal "first-best" optimum. This is because the ideal solution may violate the incentive-compatibility condition (6) for some types of agents, as was illustrated in the introduction. Specifically, note from (10) that

$$(11) \quad y^{*'}(t) = \frac{-S_{yt}(y^*(t), t)}{S_{yy}(y^*(t), t)} \cong 0 \text{ as } S_{yt}(y^*(t), t) \cong 0 \text{ for } t \in [0, 1],$$

where it is also easily seen, since (10) represents a maximum, that it is always the case that $S_{yy}(y^*(t), t) < 0$. Thus, the ideal "first-best" solution will only be feasible if $S_{yt}(y^*(t), t) \geq 0$ for all types of individuals. Hence attaining the "first-best" allocation is possible in the situation where those individuals who desire the commodity most are also the ones who should have it, in the sense that they have the highest marginal social value for it.

Next consider the situation where $S_{yt}(y^*(t), t) < 0$ for some range of agent types, say $[t_0, t_1] \subset [0, 1]$. Over this interval those individuals who desire Y the most are precisely the ones who should not have it, in the sense that they have the lowest marginal social value for it.¹¹ In this case implementing the ideal "first-best" solution is not feasible since the incentive-compatibility constraint (6) will be violated, as is evident from (11). The solution to the above programming problem determining the optimal schedule for y in this situation, now denoted by $\hat{y}(t)$, is easily seen to be described

10. The derivation of the efficiency conditions presented in this paper are straightforward exercises in optimal control theory.

11. Clearly, as can be seen from (10), $S_{yt}(y^*(t), t) < 0$ if and only if

$$- \int_0^1 \epsilon_{yt}(y^*(t), t, s) f(s) ds > V_{yt}(y^*(t), t).$$

Note that, if the external effect does not depend on the type of consumer, $\epsilon_{yt} = 0$, and this condition fails, as $V_{yt} > 0$ by (2).

by the following conditions:

$$(12) \quad S_y(\hat{y}(t), t) = 0 \quad \text{for } t \in [0, \hat{t}_0) \cup (\hat{t}_1, 1],$$

where $\hat{y}'(t) \geq 0$; and

$$(13) \quad \int_{\hat{t}_0}^{\hat{t}_1} S_y(\hat{y}(\hat{t}_0), t) f(t) dt = 0,$$

where $\hat{y}'(t) = 0$ for $t \in [\hat{t}_0, \hat{t}_1]$, and $\hat{y}(\hat{t}_0) = \hat{y}(\hat{t}_1)$. Clearly, as (12) shows, the ideal first-best solution will obtain over the regions where the incentive compatibility constraint is not binding. Thus, the preceding analysis holds for these regions. Over the range of agent types, $[\hat{t}_0, \hat{t}_1] \supset [t_0, t_1]$, for which the incentive-compatibility constraint is binding, equation (13) must hold. Thus, agents with types in this interval consume the same quantity. Note that the ceiling quantity, $\hat{y}(\hat{t}_0)$, and the interval over which it applies $[\hat{t}_0, \hat{t}_1] \supset [t_0, t_1]$ are effectively chosen so that on *average* agent's marginal net social utility will be zero over this range.¹² An illustration of the type of situation being considered is provided by Figure II.

The above allocation system can be supported by a payment scheme that is not linear in quantity (i.e., a nonlinear price schedule). Specifically, let $p(y)$ be the amount charged for y units of Y . To see how this price schedule is determined, consider those regions where the incentive compatibility constraint does not bind; here, by (2), $\hat{y}(t)$ is a strictly increasing function of t guaranteeing a univalent relationship between y and t . Now, since each agent t must voluntarily pick his prescribed Y -allocation, or $y = \hat{y}(t)$, it must transpire that

$$p'(\hat{y}(t)) = V_y(\hat{y}(t), t) \quad \text{for all } t \text{ subject to } \hat{y}'(t) > 0,$$

as this condition equalizes the private marginal costs and benefits from Y -consumption. Thus, the price schedule $p(y)$ must be predicated upon the following relationship:¹³

$$(14) \quad p'(y) = V_y(y, \hat{y}^{-1}(y)), \quad \text{for all } y \text{ subject to } \hat{y}'^{-1}(y) > 0.$$

12. This point is perhaps more easily seen by noting that in the current example the above programming problem can be written as

$$\max_{y(t), \bar{y}, \hat{t}_0, \hat{t}_1} \int_0^{\hat{t}_0} S(y(t), t) f(t) dt + \int_{\hat{t}_0}^{\hat{t}_1} S(\bar{y}, t) f(t) dt + \int_{\hat{t}_1}^1 S(y(t), t) f(t) dt,$$

subject to (6) which also generates the efficiency conditions (12) and (13).

13. Note that for those allocations where quantity controls apply,

$$\lim_{\substack{x \rightarrow y \\ x < y}} p'(x) \neq \lim_{\substack{x \rightarrow y \\ x > y}} p'(x),$$

so that the function $p'(y)$ is discontinuous at such points.

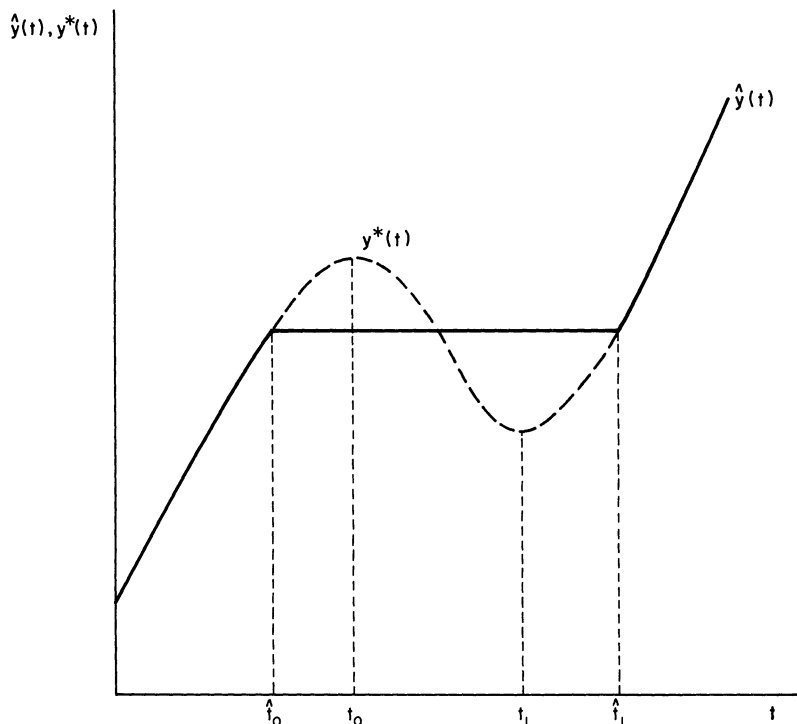


FIGURE II
Illustration of Binding Constraint

Accordingly, the price function can be written as

$$(15) \quad p(y) = p(\hat{y}(0)) + \int_{\hat{y}(0)}^y V_y(s, \hat{y}^{-1}(s)) ds$$

$$(16) \quad p(\hat{y}(0)) \equiv c \int_0^1 \hat{y}(t) f(t) dt - \int_0^1 \left[\int_0^t V_y(\hat{y}(s), s) \hat{y}'(s) ds \right] f(t) dt,$$

where the initial condition $p(\hat{y}(0))$ represents the price paid by an individual, in particular agent zero, for $\hat{y}(0)$ units of Y .¹⁴

14. The solution for the initial condition $p(\hat{y}(0))$ is determined as follows: each agent must abide by his budget constraint, implying that $p(\hat{y}(t)) + \hat{x}(t) = \bar{x}$ for all t . Thus, for agent 0, $p(\hat{y}(0)) = \bar{x} - \hat{x}(0)$. All that is needed now is an expression for $\hat{x}(0)$. To this end, note from (5') that $\hat{x}(t) = \hat{x}(0) - \int_0^t V_y(\hat{y}(s), s) \hat{y}'(s) ds$. Using this in (8) generates $\hat{x}(0) = \bar{x} + \int_0^1 \left[\int_0^t V_y(\hat{y}(s), s) \hat{y}'(s) ds - c \hat{y}(t) \right] f(t) dt$, from which the desired result is obtained.

It is interesting to note that average price, $p(y)/y$, may decline with the amount purchased, y .

Finally, suppose that $S_{y_t}(y^*(t), t) < 0$ for all t . Then a single limit \bar{y} is imposed whose level is determined by the condition $\int_0^1 S_y(\bar{y}, t) f(t) dt = 0$. The good Y will be banned outright if $\int_0^1 S_y(0, t) f(t) dt \leq 0$.¹⁵ In such an instance, when $S_{y_t}(y^*(t), t) < 0$ for all t , the social optimum can be supported by a nonlinear price schedule. However, the alternative institution of a quantity limit also implements the optimum. To see this, again consider the education example presented in the introduction. Recall that a minimum level of schooling, \bar{y} , was legislated for everyone; see Figure I. This part of society's education policy could be publicly provided and financed through general taxation by levying a fixed charge of $p(\bar{y})$ per taxpayer.¹⁶ Higher education could be purchased by citizens, on an individual basis, at the subsidized price of

$$(17) \quad p(y) - p(\bar{y}) = \int_{\bar{y}}^y V_y(s, \hat{y}^{-1}(s)) ds$$

for the *additional* $(y - \bar{y})$ units of schooling.

The point being made here is that a quantity limit *may* be consistent with social welfare maximization in the presence of asymmetric information. Such a policy could be chosen when the administrative costs of regulating externalities with it are lower than using a nonlinear price system. This, presumably, would help explain regulators' observed preference for quantity limits. The required condition, $S_{y_t} < 0$, roughly means that the individuals most desiring a commodity are the ones society least wants to have it. That is, increased desire for a quantity is correlated with increased adverse external effects, and the external loss outweighs the private gain.

III. BLACK MARKETS

What was in an individual's *ex ante* best interest is not necessarily in his *ex post* best one. While agents may have agreed

15. Gun control *may* provide an example of such a situation. Suppose that Y represents guns and t is an index of the "criminal element" in an individual. Under assumption (2) the private demand for guns is increasing in type. Given that $S_{y_t}(\hat{y}(t), t) < 0$ for all t , the more an individual desires guns the less the person should have them in the sense that he has the lowest marginal social value for them. The condition $\int_0^1 S_y(0, t) f(t) dt \leq 0$ specifies that, at any positive ceiling level for guns, the marginal social benefit from allowing desirable agents (collectors) to own guns is always outweighed by the marginal social cost from letting undesirable individuals (criminals) possess them, too. Black markets could arise, though, which operate to circumvent the government's gun control policy, an argument often used by opponents of gun control. Black markets are the subject of the next section.

16. In accordance with (16), $p(\bar{y}) = c \int_0^1 \hat{y}(t) f(t) dt - \int_0^1 [\int_0^t V_y(\hat{y}(s), s) \hat{y}'(s) ds] f(t) dt$.

upon some distribution scheme, after the allocations have in fact been made, incentives could be present for agents to trade further among themselves. That is, black markets may develop, so to speak, on which agents trade outside the contracted allocation mechanism. Such black markets operate to circumvent the original agreement. Obviously then, the potential for black markets to emerge must be taken into account when designing the optimal distribution scheme. The presence of black markets can place severe limits on the ability of the allocation system to price discriminate among agents.

An example may be the market for recreational drugs. It is widely felt that the consumption of recreational drugs by individuals generates negative externalities, due to the deleterious effects it can have on family life, schools, the workplace, and the social fabric in general. Suppose that this is true. Then in a world without black markets society's allocation mechanism could call for a prohibition on the consumption of recreational drugs. Such a policy could be thwarted, though, in a world with black markets. Now, once the presence of black markets is taken into account, society's allocation mechanism may allow some consumption of recreational drugs at regulated prices. Indeed, Friedman [1989] bases his argument for the legalization of recreational drugs, in part, on this consideration.

Suppose, for the sake of discussion, that society desires to limit the consumption of Y . Now, let there be a freely accessible black market production process where X can be transformed into Y according to $y = [1/(c + \delta)]x$. Thus, the black market price for Y is $c + \delta \equiv b$. Individuals can also freely trade unwanted quantities of Y for X among themselves on the black market at price b , but the seller of Y must incur a per unit transaction cost of δ , expressed in terms of X .¹⁷ Clearly, an optimal mechanism cannot allocate to any

17. In this discussion it is being presumed that there is an "aggregate" desire to transform X into Y . Contrarily, one could instead assume that in aggregate agents desire to transform Y into X . But this story turns out to be symmetric to the one provided in the text. Here one could let Y be transformed into X according to the black market production process $x = (c - \delta)y$ implying that Y will exchange for X at the price $c - \delta$. Finally, there is the borderline case to consider where in aggregate just the right amount of X and Y exist and agents just reallocated these total among themselves. Here the black market price for Y , or b , lies somewhere in the interval $[c - \delta, c + \delta]$, being determined by the condition,

$$\int_S [y(t) - y_d(b, t)] f(t) dt = \int_D [y_d(b, t) - y(t)] f(t) dt$$

with $x_d(\cdot)$, $y_d(\cdot) = \operatorname{argmax} \{x_d + V(y_d, t) \text{ subject to } x_d + by_d \leq x(t) + by(t), \text{ and } x_d + (b - \delta)y_d \leq x(t) + (b - \delta)y(t)\}$, and where $S = \{t | y_d(\cdot) < y(\cdot)\}$ and $D = \{t | y_d(\cdot) \geq y(\cdot)\}$. This borderline case has been abstracted from in the text.

agent, say t , an amount of good Y , or $y(t)$, which would result in his marginal valuation of this good, $V_y(y(t), t)$, exceeding the black market price for it, $b = c + \delta$. Otherwise, the agent simply purchases more Y on the black market until his marginal valuation was equated to the black market marginal cost of production. It would have been better for the distribution scheme to have provided the individual with this extra quantity of Y directly, since it could have been produced at a lower per unit cost c , rather than b . Moreover, the allocation mechanism cannot assign a quantity $y(t)$ of Y to agent t which would result in his marginal valuation for it, $V_y(y(t), t)$, falling below the marginal cost of producing it, c . If this occurred, the agent would profitably sell his Y for X on the black market until his marginal valuation for it was equal to his *net* black market selling price, $b - \delta = c$.

Therefore, a constraint on the design of the distribution system is that $y(t)$ must be chosen such that $c \leq V_y(y(t), t) \leq b \equiv c + \delta$ for all $t \in [0, 1]$. This condition can be reformulated directly in terms of bounds on $y(t)$, or on the quantity of Y that is allocated to agent t , as is shown below:

$$(18) \quad y_b(t) \leq y(t) \leq y_c(t), \quad \text{where } V_y(y_b(t), t) = b \text{ and } V_y(y_c(t), t) = c.$$

In the subsequent analysis it will be assumed that there exist intervals of agent types over which the above constraint precludes the first-best optimal allocation, $y^*(t)$, from being feasible. Specifically, let $y^*(t) > y_c(t)$ for $t \in [0, t'_0)$, and $y^*(t) < y_b(t)$ for $t \in (t'_1, 1]$.

The optimal allocation scheme in the presence of black markets, denoted by $\tilde{y}(t)$, is given by the solution to the following programming problem:

$$(19) \quad \max_{y(t)} W = \int_0^1 S(y(t), t) f(t) dt,$$

subject to (6) and

$$(20) \quad y_b(t) \leq y(t) \leq y_c(t).$$

Once again it proves useful to analyze the above problem's solution for two special cases. First, suppose that $S_{y,t}(y^*(t), t) \geq 0$ for all t . The conditions governing the optimal allocation rule for Y , or $\tilde{y}(t)$, are then easily determined to be

$$\tilde{y}(t) = \begin{cases} y_c(t) & \text{for } t \in [0, t'_0) \\ y^*(t) & \text{for } t \in [t'_0, t'_1] \\ y_b(t) & \text{for } t \in (t'_1, 1] \end{cases}, \quad \text{where note that } y^*(t'_0) = y_c(t'_0), \\ \text{and } y^*(t'_1) = y_b(t'_1).$$

An illustration of the resulting $\tilde{y}(t)$ schedule is provided in Figure III. The above allocation scheme can be supported by the nonlinear pricing system $\tilde{p}(y)$, where

$$\tilde{p}'(y) = \begin{cases} c & y < y^*(t'_0) \\ V_y(y, y^{*-1}(y)) & y = y^*(t), t \in [t'_0, t'_1] \\ b & y > y^*(t'_1), \end{cases}$$

which is portrayed by Figure IV. As can be seen, the presence of a black market limits the range of price discrimination permissible by the allocation system. Note that as the cost of transacting on the black market approaches zero, the distribution mechanism breaks down in the sense that each agent will end up consuming the quantity of Y that he would in a standard competitive equilibrium.

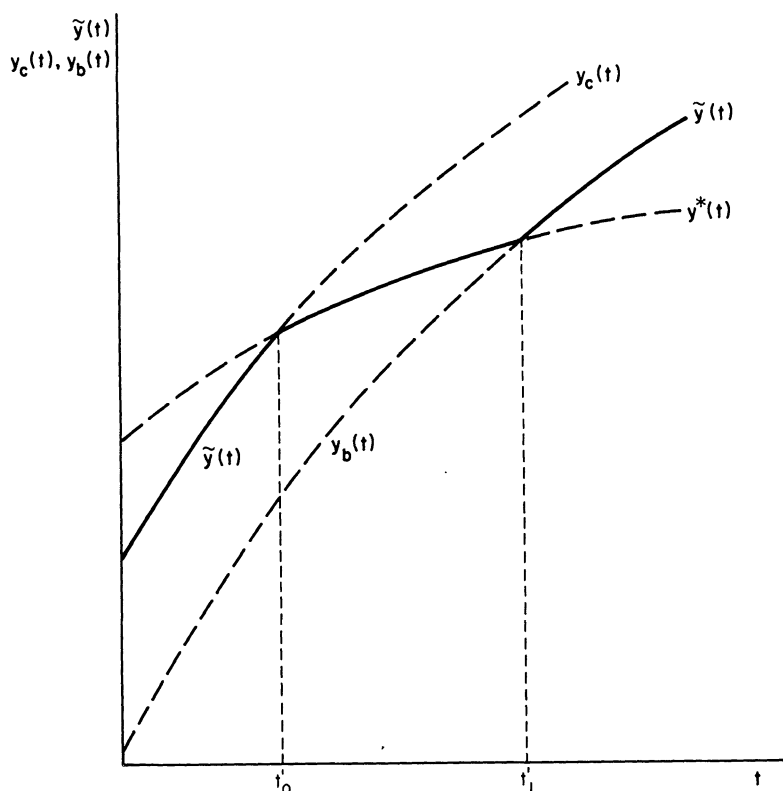


FIGURE III
Effect of Black Market

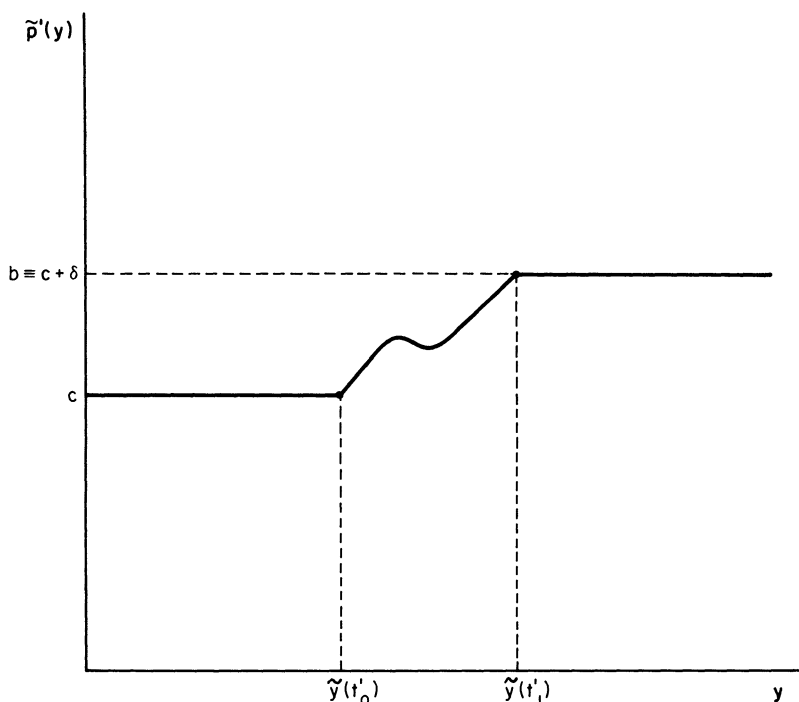


FIGURE IV
Effect of Black Market on Shadow Prices

(See Haubrich [1988] for a parallel discussion about multilateral incentive compatibility undertaken within the context of a simple intertemporal exchange economy.) It may be the case that the cost of black market transacting can be influenced to some extent by the choice of an enforcement mechanism. An increase in δ partially relaxes the constraint (18), since $dy_b(t)/d\delta < 0$, and consequently increases ex ante welfare W . Given that enforcement mechanisms are costly, the marginal benefit from doing this should be equated to the marginal cost of stricter enforcement.

Second, consider the case where $S_{yt}(y^*(t), t) < 0$ for all t . The solution for $y(t)$ is now described by the following conditions:

$$\tilde{y}(t) = \begin{cases} y_c(t) & \text{for } t \in [0, \tilde{t}_0] \\ y_c(\tilde{t}_0) & \text{for } t \in [\tilde{t}_0, \tilde{t}_1] \supset [t'_0, t'_1] \\ \text{such that } \int_{\tilde{t}_0}^{\tilde{t}_1} S_y(y_c(\tilde{t}_0), t) f(t) dt = 0 \\ y_b(t) & \text{for } t \in (\tilde{t}_1, 1], \text{ where } y_b(\tilde{t}_1) = y_c(\tilde{t}_0). \end{cases}$$

The solution is characterized by the imposition of a quantity limit $y_c(\bar{t}_0)$ on the consumption of Y over the range of agent types $[\bar{t}_0, \bar{t}_1] \supset [\bar{t}'_0, \bar{t}'_1]$, since the incentive-compatibility condition (6) is a binding constraint here. (See Figure V.) The pricing scheme, $\bar{p}(y)$, corresponding to this allocation system is

$$\bar{p}'(y) = \begin{cases} c & y < \bar{y}(\bar{t}_0) \\ b & y \geq \bar{y}(\bar{t}_0). \end{cases}$$

This pricing schedule is graphed by Figure VI. It is interesting since it shows that the allocation mechanism generates what appears to be a two-part pricing scheme. Those agents purchasing a quantity y less than $y_c(\bar{t}_0)$ pay a fixed per unit price of c , while those buying more than $y_c(\bar{t}_0)$ pay a fixed per unit price of $b = c + \delta$ on the additional $y - y_c(\bar{t}_0)$ units purchased.

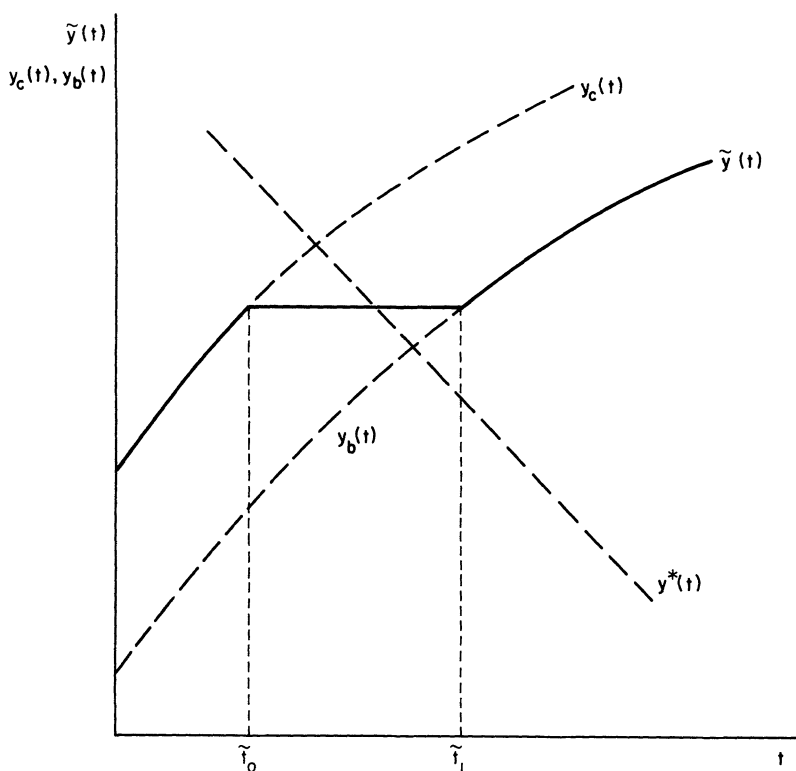


FIGURE V
Effect of Black Market When $S_{yt} < 0$

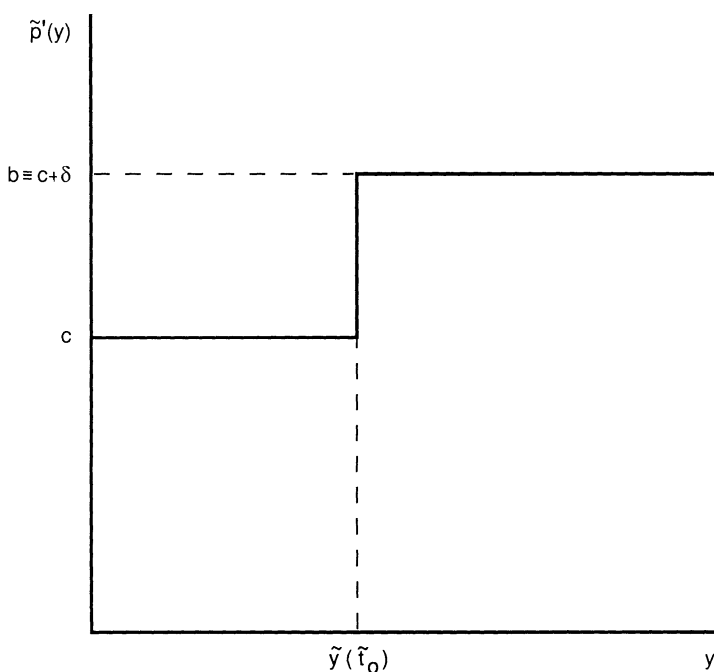


FIGURE VI

The existence of a black market generates two-part pricing when $S_{yt} < 0$.

Finally, to conclude this section, Table I is presented which summarizes the main features of the model developed.

IV. CONCLUSIONS

An analysis of the externalities problem in environments with asymmetric information was undertaken. In the full information

TABLE I

	$S_y(y^*(t), t) \geq 0$	$S_y(y^*(t), t) < 0$
No black market	—Socially efficient quantities —Taxed according to type	—Quantity limits imposed —Not taxed according to type
Black market	—Limited ability to vary opti- mally quantities according to type —Limited ability to tax accord- ing to type	—Quantity limits imposed —Two-part pricing scheme

world of Pigou, externalities are regulated via tax-cum-subsidy schemes. By contrast, in settings with asymmetric information, quantity limits may implement efficient allocations in the presence of externalities. This conclusion emerges as a direct consequence of the incentive-compatibility constraint placed on the design of the allocation mechanism. Loosely speaking, quantity limits should be imposed on an economic activity in the circumstance where the individuals who most (least) desire to engage in it are the ones whose participation is least (most) socially desirable. Finally, when consumption is unobservable, incentives are present for black markets to emerge outside of the arranged allocation system. It is clear that an allocation scheme, when being conceived, should take into account the potential for black markets to develop. This possibility can severely limit the amount of price discrimination, or variation in shadow value of consumption across agents, that mechanisms can support.

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