6

Accounting for Growth

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6.1 Introduction

The story of technological progress is the invention and subsequent implementation of improved methods of production. All models of growth incorporate this notion in some way. For example, the celebrated Solow (1956) model assumes that technological progress and its implementation are both free. Technological progress rains down as manna from heaven and improves the productivity of all factors of production, new and old alike.

Based on his earlier model, Solow (1957) proposed what has since become the dominant growth-accounting framework. Its central equation is $y = zF(k, l)$, where y is output, k and l are the quality-uncorrected inputs of capital (computed using the perpetual inventory method) and labor, and z is a measure of the state of technology. If k and l were homogenous, then this would be the right way to proceed. In principle, the framework would allow one to separate the contribution of what is measured, k and l, from what is not measured, z. Now, neither k nor l is homogenous in practice, but one could perhaps hope that some type of aggregation result would validate the procedure—if not exactly, then at least as an approximation.

The problem with this approach is that it treats all vintages of capital (or for that matter labor) as alike. In reality, advances in technology tend to be embodied in the latest vintages of capital. This means that new capital is better than old capital, not just because machines suffer wear and

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tear as they age, but also because new capital is better than the old capital was when the *latter* was new. It also means that there can be no technological progress without investment. If this is what the "embodiment of technology in capital" means, then it cannot be captured by the Solow (1956, 1957) framework, for reasons that Solow (1960, 90) himself aptly describes:

> It is as if all technical progress were something like time-and-motion study, a way of improving the organization and operation of inputs without reference to the nature of the inputs themselves. The striking assumption is that old and new capital equipment participate equally in technical progress. This conflicts with the casual observation that many if not most innovations need to be embodied in new kinds of durable equipment before they can be made effective. Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by the replacement of old-fashioned equipment by the latest models . . .

In other words, in contrast to Solow (1956, 1957), implementation is not free. It requires the purchase of new machines. Moreover, it requires new human capital, too, because workers and management must learn the new technology. This will take place either through experience or through training, or both. This type of technological progress is labelled here as *investment specific;* you must invest to realize the benefits from it.

If this view is correct, growth accounting should allow for many types of physical and human capital, each specific at least in part to the technology that it embodies. In other words, accounting for growth should proceed in a vintage capital framework. This paper argues that a vintage capital model can shed light on some key features of the postwar growth experience of the United States. The well-known models of Lucas (1988) and Romer (1990) do not fit into this framework. In Lucas's model, all physical capital, new and old alike, "participates equally" in the technological progress that the human capital sector generates; and, as Solow's quote emphasizes, this does not fit in with casual observation about how technological progress works. In contrast, Romer's model *is* a vintage capital model. New capital goods are invented every period—but new capital isn't better than old capital. It simply is different and expands the menu of available inputs, and production is assumed to be more efficient when there is a longer menu of inputs available. Thus capital does not become obsolete as it ages—an implication that denies the obvious fact that old technologies are continually being replaced by new ones.

6.1.1 Summary of the Argument and Results

Different variants of Solow's (1960) vintage capital model are explored here. To begin, however, a stab is made at accounting for postwar U.S. growth using the standard Solow (1957) framework.

Why the Model $y = zF(k, l)$ Is Unsatisfactory

Solow's (1957) model is the dominant growth-accounting framework, and section 6.2 uses it in a brief growth-accounting exercise for the postwar period. The bottom line is that this model is unable to deal with these four facts:

1. *The prolonged productivity slowdown that started around 1973.* To explain the slowdown the model insists that technological progress has been dormant since 1973! This, of course, is greatly at odds with casual empiricism: personal computers, cellular telephones, robots, and the Internet, inter alia.
2. *The falling price of capital goods relative to consumption goods.* This price has declined by 4 percent per year over the postwar period, and it is a symptom of the obsolescence of old capital caused by the arrival of better, new capital. This relative price decline of capital is not consistent with a one-sector growth model such as Solow's (1956, 1957).
3. *The productivity of a best-practice plant is much larger than that of the average plant.* They can differ by a factor of two, three, or more, depending on the industry. This is at odds with a model such as Solow's (1956, 1957), in which all firms use the same production function.
4. *The recent rise in wage inequality.* The framework is silent on this.

Why the Baseline Vintage Capital Model Is Unsatisfactory

Section 6.3 introduces the baseline vintage capital model of Solow (1960), in which technological progress is exogenous and embodied in the form of new capital goods. Using the price of new equipment relative to consumption, the technological improvement in equipment is estimated to be 4 percent per annum during the postwar period. This makes the effective capital stock grow faster than it does in conventional estimates. As a consequence, the implied productivity slowdown after 1973 is even bigger than the estimate obtained from the Solow (1957) framework! This spells trouble for frameworks that identify total factor productivity (TFP) as a measure of technological progress, a datum that Abramovitz once labelled "a measure of our ignorance." Can Solow's (1960) framework rationalize the slowdown?

Adding Diffusion Lags and Technology-Specific Learning to the Baseline Vintage Model

One adjustment to the vintage capital model that can produce a productivity slowdown is the introduction of a technology-specific learning curve on the part of users of capital goods. The effects of learning can be amplified further if spillovers in learning among capital goods users are added.

Another important adjustment is to include lags in the diffusion of new technologies. The analysis assumes that the vintage-specific efficiency of investment starts growing faster in the early 1970s with the advent of information technologies, and that the new technologies have steep learning curves. Furthermore, it is presumed that it takes some time for these technologies to diffuse through the economy. This leads to a vintage capital explanation of the "productivity slowdown" as a period of above-normal unmeasured investment in human capital specific to the technologies that came on-line starting in the early 1970s.

Implications for Wage Inequality

The productivity slowdown was accompanied by a rise in the skill premium. It is highly probable that the two phenomena are related, and section 6.5 explains why. There are two kinds of explanations for the recent rise in inequality. The first, proposed by Griliches (1969), emphasizes the role of skill in the *use* of capital goods, and is labelled "capital-skill complementarity." The second hypothesis, first proposed by Nelson and Phelps (1966), emphasizes the role of skill in *implementing* the new technology, and is labelled "skill in adoption." Both explanations can be nested in a vintage capital model.

Endogenizing Growth in the Vintage Model

Section 6.6 presents three models in which growth is endogenous, each based on a different engine of growth. Each engine requires a different fuel to run it. To analyze economic growth one needs to know what the important engines are; each one will have different implications for how resources should be allocated across the production of current and future consumptions.

Learning by doing as an engine. Section 6.6.1 describes Arrow's (1962) model of growth through learning by doing in the capital goods sector. Learning by doing is the engine that fits most closely with Solow's (1960) original vintage capital model because the technological growth that it generates uses no resources. That is, all employed labor and capital are devoted to producing either capital goods or consumption goods. As capital goods producers' efficiency rises, the relative price of capital goods falls.

Research as an engine. Section 6.6.2 highlights Krusell's (1998) model of R&D in the capital goods sector. Here each capital goods producer must decide how much labor to hire to increase the efficiency of the capital good he sells.

Human capital as an engine. Section 6.6.3 assumes that capital goods producers can switch to a better technology if they accumulate the requisite technology-specific expertise. The section extends Parente's (1994) model in which the cost of raising one's productive efficiency is the output foregone while the new technology is brought up to speed through learning.

What does the power system look like? These three models have a common structure: Each has a consumption-goods and a capital-goods sector, and each has endogenous technological progress in the capital goods sector only. This technological progress is then passed onto final output producers in the form of a "pecuniary external effect" transmitted by the falling relative price of capital. Each model focuses exclusively on one growth engine, however; and while this simplifies the exposition, it does not convey an idea of how much each engine matters to growth as a whole.

Unless its discovery was accidental, whenever a new technology appears on society's menu, society pays an invention cost. Then, society must pay an implementation cost—the cost of the physical and human capital specific to the new technology. Society needs to pay an invention cost only once per technology, whereas the implementation cost must be paid once per user.[1] After this, there are only the costs of using the technology—"production costs." Not surprisingly, then, society spends much less on research than it does on the various costs of implementing technologies. Even in the United States, Jovanovic (1997) has estimated that implementation costs outweigh research costs by a factor of about 20:1.

Because people must learn how to use new technologies, it follows that the learning costs associated with the adoption of such technologies—be they in the form of schooling, experience, or on-the-job training—are inescapable at the level of a country. Because the object of this exercise is accounting for growth in the United States, one can conclude that schooling, experience, and training are, in some combination, essential for growth to occur.[2] Research, on the other hand, clearly is not, because the majority of the world's nations have grown not by inventing their own technologies, but by implementing technologies invented by others. Presumably, the United States could do the same (assuming that other countries would then be advancing the frontiers of knowledge).

6.1.2 Why Models Matter for Growth Accounting

In its early days, the Cowles Commission's message was that aside from satisfying one's intellectual curiosity about how the world works, economic models would, on a practical level, (a) allow one to predict the consequences of out-of-sample variation in policies and other exogenous vari-

1. The average cost of implementation may, however, be declining in the number of users because of synergies in adoption.

2. Jovanovic and Rob (1998) compare Solow's (1956, 1960) frameworks against the backdrop of cross-country growth experience.

ables; (b) guide the measurement of variables; and (c) allow one to deal with simultaneity problems. These points apply to economic models generally, and they certainly relate to the value of models that explain growth. It is worth explaining why.

Policy analysis. Denison (1964, 90) claimed that "the whole embodied question is of little importance for policy in the United States." He based this assertion on his calculation that a one-year change in the average age of capital would have little impact on the output. This misses the point. Different models will suggest different growth-promoting policies. For instance, in the version of Arrow's (1962) learning-by-doing model presented here, there are industry-wide spillover effects in capital goods production, and a policy that subsidized capital goods production would improve welfare. In Parente's (1994) model, however, capital goods producers fully internalize the effects of any investment in technology-specific expertise. Such a world is efficient. Government policies may promote growth, but only at the expense of welfare. Other policy questions arise. Vintage capital models predict a continual displacement of old technologies by more efficient new ones. If a worker needs to train to work a technology, then as a technology becomes obsolete so does the worker. This may have implications for such things as unemployment. These considerations had, long ago, led Stigler to conclude that job insecurity is the price that society must pay for progress.[3]

The measurement of variables. Economic theory provides a guide about which things should be measured and how to measure them. For example, the baseline vintage capital model developed here suggests that the decline in the relative price of new equipment can be identified with the pace of technological progress in the equipment goods sector. It also provides guidance on how the aggregate stock of equipment should be measured—and this stock grows more quickly than the corresponding National Income and Product Accounts (NIPA) measure. More generally, in a world with investment-specific technological progress, new capital goods will be more productive than old ones. The rental prices for new and old capital goods are indicators of the amount of investment-specific technological progress. For example, the difference in rents between old and new office buildings (or the rent gradient) can be used to shed light on the amount of technological progress that there has been in structures.

Investment in physical capital is counted in the NIPA, whereas invest-

3. "We should like to have both a rapid increase in aggregate output and stability in its composition—the former to keep pace with expanding wants, and the latter to avoid the losses of specialized equipment of entrepreneurs and crafts of employees and creating 'sick' industries in which resources are less mobile than customers. It is highly probable that the goals are inconsistent" (Stigler 1947, 30).

ment in knowledge is not. Yet, investment in knowledge may increase output tomorrow in just the same way as investment in physical capital does. This is sometimes referred to as the unmeasured investment problem. In the United States, R&D spending amounts to about 3 percent of GDP. The costs of implementing new technologies, in terms of schooling, on-the-job training, and so on, may amount to 10 percent of GDP. The models of Krusell (1998) and Parente (1994) suggest that such expenditures are as vital to the production of future output as is investment in equipment and structures. In the NIPA such expenditures are expensed or deducted from a firm's profits, as opposed to being capitalized into profits as when a firm makes a new investment good. By this accounting, GDP would be 13 percent higher if these unmeasured investments were taken into account.

This, indeed, is one way the vintage capital model can perhaps explain the productivity slowdown—the vintage of technologies that arrived around 1974 was promising but was subject to a protracted learning curve and high adoption costs. The productivity slowdown took place, in other words, because there was a lot of unmeasured investment. Conventional growth accounting practices will understate productivity growth to the extent that they underestimate output growth due to these unmeasured investments. This might suggest that more effort should be put into collecting aggregate data on R&D and adoption costs.

Simultaneity problems. Conventional growth accounting uses an aggregate production function to decompose output growth into technological progress and changes in inputs in a way that uses minimal economic theory. Clearly, though, a large part of the growth in the capital stock—equipment and structures—is due to technological progress. The general equilibrium approach taken here allows for the growth in capital stock to be broken down into its underlying sources of technological progress. Furthermore, it links the observed decline in the price of new equipment with the rate of technological progress in the production of new equipment. More generally, models allow one to connect observed rent gradients on buildings to the rate of technological progress in structures, and they allow one to connect the long diffusion lags of products and technologies to the costs of adopting them. Models lead to more precise inferences about such simultaneities.

6.2 Solow (1957) and Neutral Technological Progress

In one of those rare papers that changes the courses of economics, Solow (1957) proposed a way of measuring technological progress. Suppose output, y, is produced according to the constant-returns-to-scale production function

$$y = zF(k,l), \tag{1}$$

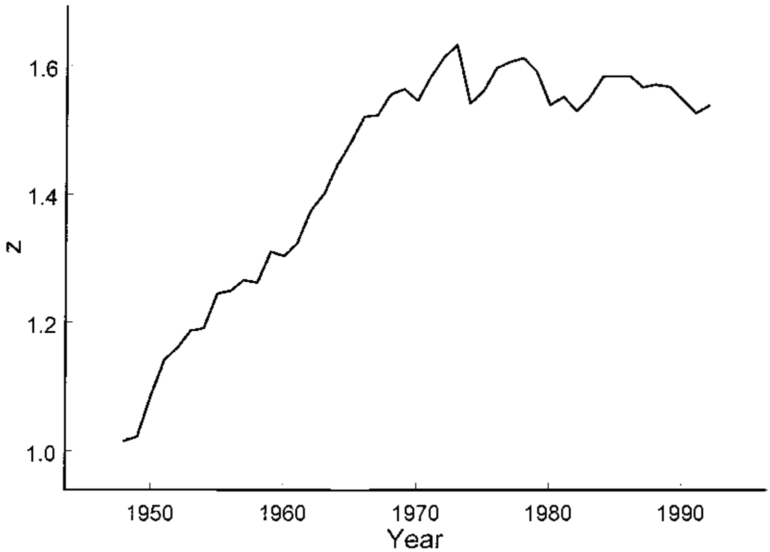

Fig. 6.1 Standard measure of neutral technological progress

where k and l are the inputs of capital and labor. The variable z measures the state of technology in the economy, and technological progress is neutral. Over time, z grows, reflecting technological improvement in the economy. Thus, for a given level of inputs, k and l, more output, y, can be produced.

For any variable x, let $g_x \equiv (1/x)(dx/dt)$ denote its rate of growth. If the economy is competitive, then the rate of technological progress can be measured by

$$g_z = g_{y/l} - \alpha g_{k/l}, \tag{2}$$

where α represents capital's share of income.

The rate of technological progress, g_z, can easily be computed from equation (2), given data on GDP, y, the capital stock, k, hours worked, l, and labor's share of income, $1 - \alpha$. Figure 6.1 plots z for the postwar period. Note that the growth in z slows down dramatically around 1973.[4] This is often referred to as the "productivity slowdown." Does it seem reasonable to believe that technological progress has been dormant since 1973? Hardly. Casual empiricism speaks to the contrary: computers, robots, cellular telephones, and so on.

Perhaps part of the explanation is that some quality change in output goes unmeasured so that g_y was understated. However, the above measures

4. In fact, over the whole period it grew on average at the paltry rate of 0.96 percent per year.

of k and l do not control for quality change, and this biases things in the other direction and makes the puzzle seem even larger. Is something wrong with the notion of technological progress in the Solow (1957) model? The remaining sections analyze vintage capital models in which technological progress is investment specific.

6.3 Solow (1960) and Investment-Specific Technological Progress

In a lesser-known paper, Solow (1960) developed a model that embodies technological progress in the form of *new* capital goods.

The production of final output. Suppose that output is produced according to the constant-returns-to-scale production function

$$y = F(k, l). \tag{3}$$

Note that there is no neutral technological progress. Output can be used for two purposes: consumption, c, and gross investment, i. Thus, the economy's resource constraint reads: $c + i = F(k, l)$

Capital accumulation. Now, suppose that capital accumulation is governed by the law of motion

$$\frac{dk}{dt} = iq - \delta k, \tag{4}$$

where i is gross investment and δ is the rate of physical depreciation on capital. Here q represents the current state of technology for producing new equipment. As q rises more new capital goods can be produced for a unit of forgone output or consumption. This form of technological progress is specific to the investment goods sector of the economy. Therefore, changes in q are dubbed *investment-specific* technological progress. Two important implications of equation (4) are:

1. In order to realize the gains from this form of technological progress there must be investment in the economy. This is not the case for neutral technological progress, as assumed in Solow (1957).
2. Efficiency units of capital of different vintages can be aggregated linearly in equation (3) using the appropriate weights on past investments: $k(t) = \int_0^\infty e^{-\delta s} q(t - s) i(t - s) ds$.[5]

The relative price of capital. In a competitive equilibrium the relative price of new capital goods, p, would be given by $p = 1/q$, because this shows

5. Benhabib and Rustichini (1991) relax this assumption and allow for a variable rate of substitution in production between capital stocks of different vintages.

how much output or consumption goods must be given up in order to purchase a new unit of equipment. Therefore, in the above framework it is easy to identify the investment-specific technological shift factor, q, by using a price series for new capital goods—that is, by using the relationship $q = 1/p$.

Growth accounting in the baseline model. Figure 6.2 shows the price series for new equipment and the implied series for the investment technology shock. Look at how much better this series represents technological progress. It rises more or less continuously throughout the postwar period; there is no productivity slowdown here.

So how much postwar economic growth is due to investment-specific versus neutral technological progress? To gauge this, assume that output is given by the production function

$$y = zk_e^{\alpha_e}k_s^{\alpha_s}l^{1-\alpha_e-\alpha_s}, \tag{5}$$

where k_e and k_s represent the stocks of equipment and structures in the economy. Let equipment follow a law of motion similar to equation (4) so that

$$\frac{dk_e}{dt} = qi_e - \delta_e k_e, \tag{6}$$

where i_e is gross investment in equipment measured in consumption units and δ_e is the rate of physical depreciation. Thus, equipment is subject to

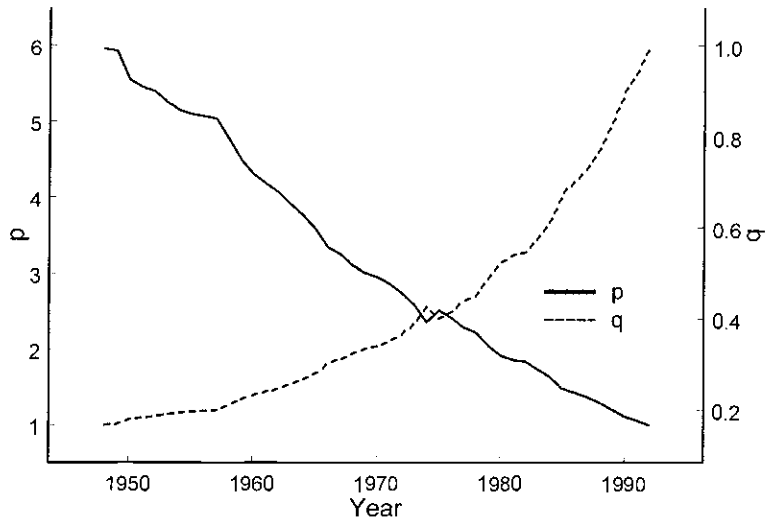

Fig. 6.2 Relative price of equipment, *p*, and investment-specific technological progress, *q*

investment-specific technological progress. The law of motion for structures is written as

$$\text{(7)} \qquad \frac{dk_s}{dt} = i_s - \delta_s k_s,$$

where i_s represents gross investment in structures measured in consumption units and δ_s is the rate of physical depreciation. The economy's resource constraint now reads[6]

$$\text{(8)} \qquad c + i_e + i_s = y.$$

It is easy to calculate from equation (5), in conjunction with equations (6)–(8), that along the economy's balanced path the rate of growth in income is given by

$$\text{(9)} \qquad g_y = \left(\frac{1}{1 - \alpha_e - \alpha_s}\right) g_z + \left(\frac{\alpha_e}{1 - \alpha_e - \alpha_s}\right) g_q.$$

To use this formula, numbers are needed for α_e, α_s, g_z, and q_q. Let $\alpha_e = 0.17$ and $\alpha_s = 0.13$.[7] Over the postwar period the rate of investment-specific technological progress averaged 4 percent per year, a fact that can be computed from the series shown for q in figure 6.2. Hence, $g_q = 0.04$. Next, a measure for z can be obtained from the production relationship in equation (5) that implies $z = (y/[k_e^{\alpha_e} k_s^{\alpha_s} l^{1-\alpha_e-\alpha_s}])$. Given data on y, k_e, k_s, and l the series for z can be computed. These series are all readily available, except for k_e. This series can be constructed using the law of motion in equation (6) and data for q and i_e. In line with NIPA, the rate of physical depreciation on equipment was taken to be 12.4 percent, so that $\delta_e = 0.124$. Following this procedure, the average rate of neutral technological advance was estimated to be 0.38 percent. Equation (9) then implies that investment-specific technological progress accounted for 63 percent of output growth, whereas neutral technological advance accounted for 35 percent.[8]

Why the baseline model is not adequate. All is not well with this model, however. With the quality change correction in the capital stock, it grows faster than did k in the Solow (1957) version. When this revised series is inserted into the production function for final goods, the implied produc-

6. Greenwood, Hercowitz, and Krusell (1997) have used this structure for growth accounting. Hulten (1992) employs a similar setup but replaces the resource constraint with $c + i_e q + i_s = y$. See Greenwood, Hercowitz, and Krusell (1997) for a discussion on the implications of this substitution.

7. This is what Greenwood, Hercowitz, and Krusell (1997) estimated.

8. The second part of the appendix works out what would happen if a growth accountant failed to incorporate investment-specific technological advance into his analysis.

tivity slowdown is even bigger than that arising from the Solow (1957) framework. How can this slowdown be explained? The introduction of lags in learning about how to use new technologies to their full potential, and lags in the diffusion of new technologies, seems to do the trick.

6.4 Adjusting the Baseline Solow (1960) Model

This section introduces lags in learning and diffusion of new technologies. The setting is necessarily one in which plants differ in the technologies they use. As will be seen, it turns out that aggregation to a simple growth model cannot be guaranteed in such settings. Some conditions on technology and the vintage structure that ensure Solow (1960) aggregation are presented.

6.4.1 Heterogeneity across Plants and the Aggregation of Capital

Notation. Some of the following variables are plant specific. Because plants of different ages, τ, will coexist at any date, one sometimes needs to distinguish these variables with a double index. The notation $x_\tau(t)$ will denote the value of the variable x at date t in a plant that is τ years old. The plant's vintage is then $v = t - \tau$. Variables that are not plant specific will be indexed by t alone. Moreover, the index t will be dropped whenever possible.

Production of final goods. Final goods are produced in a variety of plants. A plant is indexed by its vintage. Thus, the output of a plant of age τ is described by the production function

$$y_\tau = z_\tau k_\tau^\alpha l_\tau^\beta, \quad 0 < \alpha + \beta < 1,$$

where z_τ is the plant's TFP and k_τ and l_τ are the stocks of capital and labor that it employs. For now, z_τ is exogenous. A plant's capital depreciates at the rate δ and cannot be augmented once in place.

Investment-specific technological progress. Recall that g_q is the rate of investment-specific technological progress. Then, as before, an efficiency unit of new capital costs $1/q(t) = p(t) = e^{-g_q t}$ units of consumption in period t. The period t cost of the capital for new plant is therefore $k_0(t)/q(t)$.

Optimal hiring of labor. A price-taking plant of age τ will hire labor up to the point where the marginal product of labor equals the wage, w. Hence, $\beta z_\tau k_\tau^\alpha l_\tau^{\beta-1} = w$, so that

$$l_\tau = \left(\frac{\beta z_\tau k_\tau^\alpha}{w}\right)^{1/(1-\beta)}, \tag{10}$$

and

$$(11) \qquad y_\tau = \left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} z_\tau^{1/(1-\beta)} k_\tau^{\alpha/(1-\beta)}.$$

Labor market clearing. Suppose that there are n_τ plants operating of age τ. If the aggregate endowment of labor is fixed at h, then labor market clearing requires that

$$\int_0^\infty n_\tau l_\tau d\tau = h.$$

Substituting equation (10) into the above formula then allows the following expression to be obtained for the market clearing wage

$$(12) \qquad w = \beta\left[\frac{\int_0^\infty n_\tau (z_\tau k_\tau^\alpha)^{1/(1-\beta)} d\tau}{h}\right]^{1-\beta}.$$

Plugging this into equation (11) yields the output of an age-τ plant as follows:

$$y_\tau = z_\tau^{1/(1-\beta)} k_\tau^{\alpha/(1-\beta)} \left[\frac{h}{\int_0^\infty n_\tau (z_\tau k_\tau^\alpha)^{1/(1-\beta)} d\tau}\right]^\beta.$$

Aggregate output. Aggregate output is the sum of outputs across all the plants: $y = \int_0^\infty n_\tau y_\tau d\tau$. It therefore equals

$$(13) \qquad y = \frac{h^\beta \int_0^\infty n_\tau z_\tau^{1/(1-\beta)} k_\tau^{\alpha/(1-\beta)} d\tau}{\left[\int_0^\infty n_\tau (z_\tau k_\tau^\alpha)^{1/(1-\beta)} d\tau\right]^\beta} = h^\beta \left(\int_0^\infty n_\tau z_\tau^{1/(1-\beta)} k_\tau^{\alpha/(1-\beta)} d\tau\right)^{1-\beta}.$$

Solow (1960) aggregation. This model is similar to the benchmark vintage capital model. In fact, it aggregates to it exactly if the following three assumptions hold:

1. Returns to scale are constant (so that $\alpha = 1 - \beta$).
2. Total factor productivity is the same in all plants (so that $z_\tau = z$).
3. The number of plants of each vintage does not change over time. That is, $n_{t-v}(t) = n_0(v)$, or equivalently, $n_\tau(t) = n_0(t - \tau)$, since $v = t - \tau$. In other words, all investment is in the current vintage plants, and plants last forever—only their capital wears off asymptotically.

In this situation, $y = zh^{1-\alpha}\mathbf{k}^\alpha$, where the aggregate capital stock $\mathbf{k}$ is defined by $\mathbf{k}(t) = \int_{-\infty}^t n_{t-v}(t) k_{t-v}(t) dv$. Now capital in each plant depreciates at the rate δ, which means that for any $v \leq t$,

$$\frac{dk_{t-v}(t)}{dt} = -\delta k_{t-v}(t).$$

Moreover, by assumption (3), $[dn_{t-v}(t)]/dt = 0$ for any $v \leq t$. Therefore,

$$\frac{d\mathbf{k}(t)}{dt} = -\delta\mathbf{k}(t) + q(t)i(t),$$

where $i(t) = [n_0(t)k_0(t)]/q(t)$ is gross investment (measured in consumption units).[9] If one identifies h and $\mathbf{k}$ with l and k in equations (3) and (4), the two models will have identical predictions.[10]

So, for the above vintage capital model to differ in a significant way from the benchmark model with investment-specific technological progress, some combination of assumptions (1), (2), and (3) must be relaxed. Without this, the model will be unable to resolve the productivity slowdown puzzle.

Lumpy investment assumption. Now, for the rest of section 6.4, suppose that the blueprints for a new plant at date t call for a *fixed* lump of capital, $k_0(t)$. Let $k_0(t)$ grow at the constant rate $\kappa \equiv g_q/(1 - \alpha)$ over time.[11] That is, the efficiency units of capital embodied by a new plant at date t are equal to $k_0(t) = e^{\kappa t}$. A plant built at date t then embodies $e^{\kappa t} \equiv (e^{g_q t})^{1/(1-\alpha)}$ efficiency units of capital. Thus, the consumption cost of building a new plant at date t is

$$\frac{e^{\kappa\tau}}{q(t)} = e^{(\kappa - g_q)t} = (e^{g_q t})^{\alpha/(1-\alpha)}. \tag{14}$$

Therefore, the ratio of the capital stock between a new plant and a plant that is τ periods old will be given by $k_0/k_\tau = e^{(\kappa+\delta)\tau}$, where δ is the rate of physical depreciation on capital. Together with equation (11) this implies that $\lim_{\tau\to\infty} y_\tau/y_0 = 0$ so that, relative to new plants, old plants will wither away over time. In what follows, set $\delta = 0$.

6.4.2 Learning Effects

Established skills are often destroyed, and productivity can temporarily fall upon a switch to a new technology. In its early phases, then, a new

9. It makes sense that under constant returns to scale only the aggregate amount of investment matters, and not how it is divided among plants.

10. Even without these assumptions, the model will behave similarly in balanced growth. Assume for the moment that the number of plants is constant through time so that $n_\tau = n$ and z_τ grows at rate g_z. The supply of labor will be constant in balanced growth. Now, along a balanced growth path, output and investment must grow at a constant rate, g_y. This implies that $i = nk_0/q$ must grow at this rate, too. Therefore, k_0 must grow at rate $g_y + g_q$. Clearly, to have balanced growth, all of the k_τ's should grow at the same rate. Consequently, k_τ will grow at rate $g_y + g_q$. It is easy to deduce from equation (13) that the rate of growth in output will be given by $g_y = (1/y)(dy/dt) = [(1/1 - \alpha)g_z] + [(\alpha/1 - \alpha)g_q]$. This formula is identical in form to equation (9). (To see this, set $\alpha_s = 0$ and $\alpha_e = \alpha$ in equation [9]).

11. From note 10, it is known that along a balanced growth path, $k_0(t)$ must grow at rate $g_y + g_q$, where g_y is the growth rate of output and g_q is the growth rate of q. It is easy to check that $g_y + g_q = g_q/(1 - \alpha) \equiv \kappa$.

technology may be operated inefficiently because of a dearth of experience.

Evidence on learning effects. A mountain of evidence attests to the presence of such learning effects.

1. An interesting case study, undertaken by David (1975), is the Lawrence no. 2 cotton mill. This mill was operated in the U.S. antebellum period, and detailed inventory records show that no new equipment was added between 1836 and 1856. Yet, output per hour grew at 2.3 percent per year over the period. Jovanovic and Nyarko (1995) present a variety of learning curves for activities such as angioplasty surgery to steel finishing; see Argotte and Epple (1990) for a survey of case studies on learning curves.
2. After analyzing 2,000 firms from forty-one industries spanning the period 1973–86, Bahk and Gort (1993) find that a plant's productivity increases by 15 percent over the first fourteen years of its life due to learning effects.

The learning curve. A simple functional form for the learning curve will now be assumed. Suppose that as a function of its age, τ, a plant's time t TFP $z_\tau(t)$ does not depend on t per se, but only on τ, as follows:

$$z_\tau = (1 - z^* e^{-\lambda\tau})^{1-\beta}.$$

Thus, as a plant ages it becomes more productive, due (for example) to learning by doing. Observe that $z_0 = (1 - z^*)^{1-\beta}$, so that $1 - (1 - z^*)^{1-\beta}$ is the "amount to be learned." Moreover, z_τ is bounded above by one so that you can only do so much with any particular technology. Times of rapid technological progress are likely to have steeper learning curves. That is, z^* is likely to be positively related to the rate of investment-specific technological progress, g_q. The bigger g_q is, the less familiar the latest generation of capital goods will look, and the more there will be with which to get acquainted. Therefore assume that

$$z^* = \omega g_q^{\nu}. \tag{15}$$

In what follows, assume that $\beta = 0.70$, $\lambda = 1.2$, $\omega = 0.3$, and $\nu = 12$. With this choice of parameter values, the learning curve shows a fairly quick rate of learning in that a plant's full potential is reached in about fifteen years (when g_q takes its postwar value of 0.04).

6.4.3 Diffusion Lags

Evidence. Diffusion refers to the spread of a new technology through an economy. The diffusion of innovations is slow, but its pace seems to be increasing over time. In a classic study Gort and Klepper (1982) examined forty-six product innovations, beginning with phonograph records in 1887 and ending with lasers in 1960. The authors traced diffusion by examining the number of firms that were producing the new product over time. On average, only two or three firms were producing each new product for the first fourteen years after its commercial development; then the number of firms sharply increased (on average six firms per year over the next ten years). Prices fell rapidly following the inception of a new product (13 percent a year for the first twenty-four years). Using a twenty-one-product subset of the Gort and Klepper data, Jovanovic and Lach (1997) report that it took approximately fifteen years for the output of a new product to rise from the 10 percent to the 90 percent diffusion level. They also cite evidence from a study of 265 innovations that found that a new innovation took forty-one years on average to move from the 10 percent to the 90 percent diffusion level. Grübler (1991) also presents evidence on how fast these products spread after they are invented. For example, in the United States the steam locomotive took fifty-four years to move from the 10 percent to the 90 percent diffusion level, whereas the diesel (a smaller innovation) took twelve years. It took approximately twenty-five years from the time the first diesel locomotive was introduced in 1925 to the time that diesels accounted for half of the locomotives in use, which occurred somewhere between 1951 and 1952.

Theories of diffusion lags. Diffusion lags seem to have several distinct origins:

1. *Vintage-specific physical capital.* If, in a vintage capital model, a firm can use just one technology at a time, as in Parente (1994), it faces a replacement problem. New equipment is costly, whereas old, inferior equipment has been paid for. Hence it is optimal to wait a while before replacing an old machine with a new, better one.[12] Furthermore, not everyone can adopt at the same time because the economy's capacity to produce equipment is finite. This implies some smoothing in adoption, and a smooth diffusion curve.

2. *Vintage-specific human capital.* The slow learning of new technologies acts to make adoption costly and slow it down, a fact that Parente (1994) and Greenwood and Yorukoglu (1997) emphasize. Adoption of a new technology may also be delayed because it is difficult at first to hire experienced people to work with them, as Chari and Hopenhayn (1991) emphasize.

12. For instance, David (1991) attributes the slow adoption of electricity in manufacturing during the early 1900s partly to the durability of old plants' use of mechanical power derived from water and steam. Those industries undergoing rapid expansion and hence rapid *net* investment—tobacco, fabricated metal, transportation, and equipment—tended to adopt electricity first.

3. *Second-mover advantages.* If, as Arrow (1962) assumes, the experience of early adopters is of help to those that adopt later, firms have an incentive to delay, and it is not an equilibrium for firms to adopt a new technology en masse; some will adopt right away, and others will choose to wait, as in models such as Jovanovic and Lach (1989) and Kapur (1993).

4. *Lack of awareness.* A firm may not be aware of any or all of the following: (a) that a new technology exists, (b) that it is suitable, or (c) where to acquire all the complementary goods. Diffusion lags then arise because of search costs, as in Jovanovic and Rob (1989) and Jovanovic and MacDonald (1994).[13]

5. *Other differences among adopters.* Given that origins 1–4 provide adopters a reason to wait, the optimal waiting time of adopters will differ simply because adopters "are different." For instance, the diffusion of hybrid corn was affected by economic factors such as the profitability of corn (relative to other agricultural goods) in the area in question, and the education of the farmers that resided there (Griliches 1957; Mansfield 1963; Romeo 1975).

Determining the number of entering plants, $n_0(t)$. To get a determinate number of plants of any vintage, the constant returns to scale assumption must be dropped. Suppose that there are diminishing returns to scale so that $\alpha + \beta < 1$. The profits from operating an age τ plant in the current period will be given by

$$\pi_\tau \equiv \max_{l_\tau}[z_\tau k_\tau^\alpha l_\tau^\beta - wl_\tau] = (1 - \beta)\left[\left(\frac{\beta}{w}\right)^\beta z_\tau k_\tau^\alpha\right]^{1/(1-\beta)}.$$

The present value of the flow of profits from bringing a new plant on line in the current period, t, will read

$$\int_0^\infty \pi_\tau(t + \tau)e^{-r\tau}d\tau - \frac{k_0(t)}{q(t)} - \phi(t),$$

where r denotes the real interest rate. From equation (14), $k_0(t)/q(t) = e^{[\alpha/(1-\alpha)]g_q t}$ is the purchase price of the newly installed capital, and $\phi(t) = \phi_0 e^{[\alpha/(1-\alpha)]g_q t}$ is the fixed cost of entry. If there is free entry into production, then these rents must be driven down to zero so that

$$\int_0^\infty \pi_\tau(t + \tau)e^{-r\tau}d\tau - \frac{k_0(t)}{q(t)} - \phi(t) = 0. \tag{16}$$

13. The diffusion of technology has steadily gotten faster over the last century (Federal Reserve Bank of Dallas 1997, exhibit D). Search-theoretic models of technological advance naturally attribute this trend to the secular improvement in the speed and quality of communication.

This equation determines the number of new entrants $n_0(t)$ in period t. Although $n_0(t)$ does not appear directly in this equation, it affects profits because through equation (12) it affects the wage.

Choosing values for α and β. In the subsequent analysis, labor's share of income will be assumed to equal 70 percent so that $\beta = 0.70$. From the national income accounts alone it is impossible to tell how the remaining income should be divided up between profits and the return to capital. Assume that capital's share of income is 20 percent, implying that $\alpha = 0.20$, so that rents will amount to the remaining 10 percent of income. The real interest rate, r, is taken to be 6 percent.

A parametric diffusion curve. In what follows, a particular outcome for the diffusion curve for new inventions is simply postulated, as in Jovanovic and Lach (1997). Consider a switch in the economy's technological paradigm that involves moving from one balanced growth path, with some constant flow of entrants n^*, toward another balanced growth path, with a constant flow of entrants n^{**}. These flows of entrants should be determined in line with equation (16). Along the transition path there will be some flow of new entrants each period. Suppose that the number of plants adopting the new paradigm follows a typical S-shaped diffusion curve. Specifically, let

$$\frac{\int_0^\infty n_0(s)ds}{tn^{**}} = \frac{1}{1 + e^{(\Delta - \varepsilon t)}}.$$

The parameter Δ controls the initial number of users, or $n_0(0)$, while ε governs the speed of adoption. Assume that $\Delta = 3.5$ and $\varepsilon = 0.15$. With this choice of parameter values, it takes approximately twenty-five years to reach the 50 percent diffusion level, or the point at which about 50 percent of the potential users (as measured by tn^{**} have adopted the new technology.

Spillover Effects in Learning a Technology

Suppose that a new technological paradigm (for instance, information technology) is introduced at date $t = 0$, for the first time. Better information technologies keep arriving, but they all fit into the new paradigm, so as each new grade is adopted, the economy gains expertise about the entire paradigm. For someone who adopts a particular technological grade from this new paradigm, the ease of learning about this particular technological grade might be related to the cumulative number of users of the paradigm itself. The more users, the easier it is to acquire the expertise to run a new technological grade efficiently. In particular, let the starting point of the diffusion curve for a particular technological grade within the new paradigm depend positively on the number of plants that have already adopted a technology from the new paradigm. This number of adopters is an increasing function of time. Hence amend equation (15) to read

$$z^* = \omega g_q^{\nu} + \chi\left[1 - \frac{1}{1 + e^{(\Delta-\varepsilon t)}}\right]^{\sigma},$$

where χ and σ are constants. Observe that z^* (a measure of the amount to be learned on one's own) is decreasing in t (the time elapsed since the first usage of the new paradigm in question). As $t\rightarrow\infty$ the spillover term vanishes. The strength of the spillover term is increasing in χ and decreasing in σ. In the subsequent analysis it will be assumed that $\chi = 0.4$ and $\sigma = 0.02$.

6.4.4 An Example: The Third Industrial Revolution

Now, imagine starting off along a balanced growth path where the rate of investment-specific technological progress is g_q^*. All of a sudden—at a point in time that will be normalized to $t = 0$—a new technological paradigm appears that has a higher rate of investment-specific technological progress, g_q^{**}. Because of the effect of g_q on learning, as specified in equation (15), learning curves become steeper once the new technological era dawns.

Perhaps the first balanced growth path could be viewed as the trajectory associated with the second industrial revolution. This period saw the rise of electricity, the internal combustion engine, and the modern chemical industry. The second event could be the dawning of the information age, or the third industrial revolution. What will the economy's transition path look like? How does this transition path depend on learning and diffusion?

For this experiment let $g_q^* = 0.035$ and $g_q^{**} = 0.05$. Figure 6.3 plots labor productivity for the economy under study. The straight line depicts what would have happened to productivity had information technology not been invented at all. The remarkable finding is how growth in labor productivity stalls during the nascent information age. Note that it takes productivity about thirty or so years to cross its old level.

The importance of learning is shown in figure 6.4, which plots the transition path when there are no learning effects. It now takes ten years less for productivity to cross its old trend path. Last, figure 6.5 shuts down the diffusion curve. There is still a productivity slowdown due to learning effects, but it is much weaker. The learning effects in the model are muted for two reasons. First, it takes no resources to learn. If learning required the input of labor, intermediate inputs, or capital, the effect would be strengthened. Second, in the model labor can be freely allocated across vintages. Therefore, less labor is allocated to the low productivity plants (such as

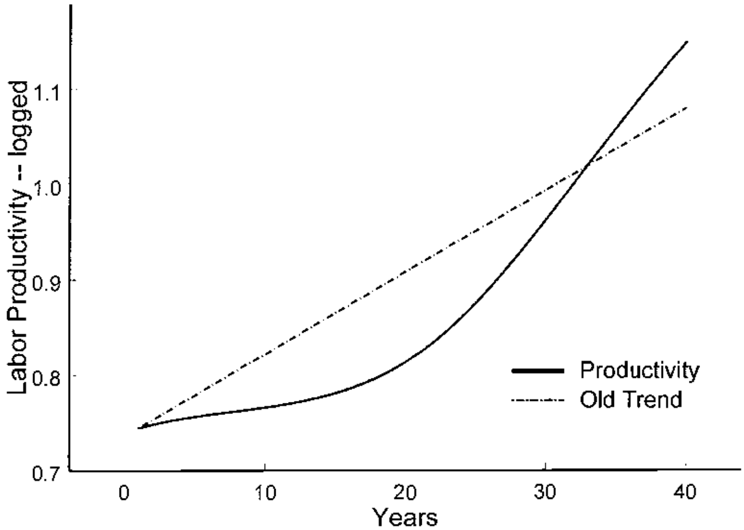

Fig. 6.3 Transitional dynamics

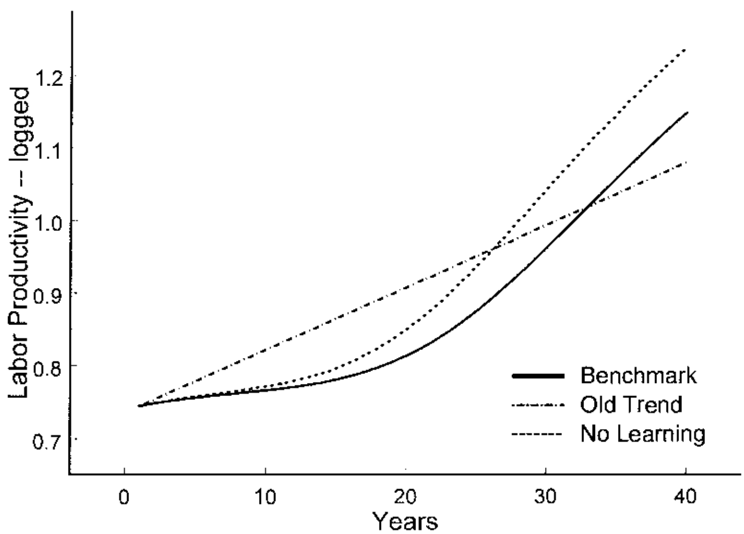

Fig. 6.4 Transitional dynamics (no learning)

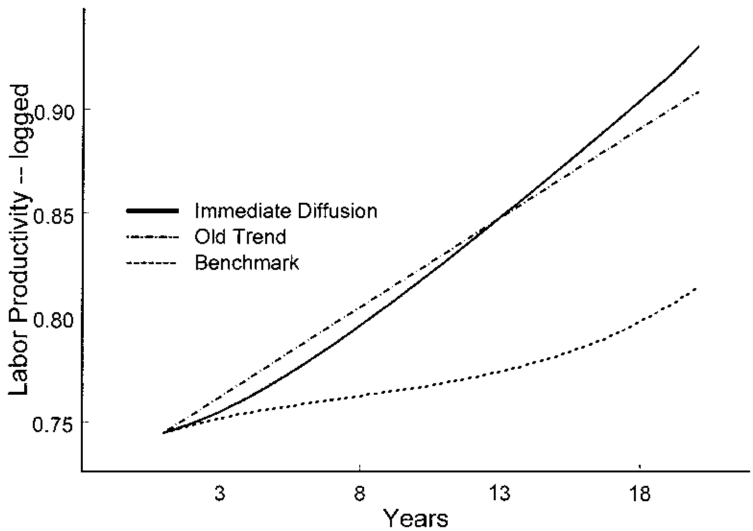

Fig. 6.5 Transitional dynamics (immediate diffusion)

the new plants coming on-line) and this ameliorates the productivity slowdown. If each plant required some minimal amount of labor to operate—another condition that would break Solow (1960) aggregation—then the learning effects would be stronger. Finally, a key reason for slow diffusion curves is high learning costs, and this channel of effect has been abstracted away from here. Learning and diffusion are likely to be inextricably linked and therefore difficult to separate, except in an artificial way, as was done here.

These figures make it clear that the vintage capital model can indeed explain the productivity slowdown if learning and diffusion lags matter enough, and the evidence presented here indicates that they do. Another appealing feature of the model is that it can also explain the concurrent rise in the skill premium, and this is the subject of the next section.

6.5 Wage Inequality

As labor-productivity growth slowed down in the early 1970s, wage inequality rose dramatically. Recent evidence suggests that this rise in wage inequality may have been caused by the introduction of new capital goods. For instance:

1. The era of electricity in manufacturing dawned around 1900. Goldin and Katz (1998) report that industries that used electricity tended to favor the use of skilled labor.

2. Autor, Katz, and Krueger (1998) find that the spread of computers may explain 30 to 50 percent of the growth in the demand for skilled workers since the 1970s.

3. Using cross-country data, Flug and Hercowitz (2000) discover that an increase in equipment investment leads to a rise both in the demand for skilled labor and in the skill premium. In a similar vein, Caselli (1999) documents, from a sample of U.S. manufacturing industries, that since 1975 there has been a strong, positive relationship between changes in an industry's capital-labor ratio and changes in its wages.

Theories of how skill interacts with new technology are of two kinds. The first kind of theory emphasizes the role of skill in the *use* of capital goods that embody technology. Here it is assumed that technology is embodied in capital goods. This is labelled the capital-skill complementarity hypothesis. The second hypothesis emphasizes the role of skill in *implementing* the new technology, referred to as skill in adoption.

6.5.1 Griliches (1969) and Capital-Skill Complementarity

The hypothesis in its original form. In its original form, the hypothesis fits in well with a minor modification of Solow (1956, 1957) that allows for two kinds of labor instead of one. Suppose, as Griliches (1969) proposed, that in production, capital is more complementary with skilled labor than with unskilled labor. Specifically, imagine an aggregate production function of the form

$$y = [\theta k^{\rho} + (1 - \theta)s^{\rho}]^{\alpha/\rho}u^{(1-\alpha)},$$

where s and u represent inputs of skilled and unskilled labor. Capital and skill are complements, in the sense that the elasticity of substitution between them is less than unity if $\rho < 0$. The skill premium, or the ratio of the skilled to unskilled wage rates, is just the ratio of the marginal products of the two types of labor:

$$\frac{\frac{\partial y}{\partial s}}{\frac{\partial y}{\partial u}} = \frac{\alpha(1 - \theta)}{1 - \alpha}\left[\theta\left(\frac{k}{s}\right)^{\rho} + (1 - \theta)\right]^{-1}\left(\frac{u}{s}\right).$$

Now, suppose that the endowments of skilled and unskilled labor are fixed. Then, the skill premium will rise whenever the capital stock increases, and so will labor's share of income.[14] Krusell et al. (2000) argue that an aggregate production function of this type fits the postwar experi-

14. Unskilled labor's share of income remains constant, whereas skilled labor's share increases.

ence well, provided that k is computed as in the benchmark vintage capital model of section 6.3: $k(t) = \int_0^\infty e^{-\delta s} q(t - s)i(t - s)ds$.

Shifts in the production structure. In Griliches's formulation, the skill premium depends on the supplies of the factors k, s, and u only. However, the premium will also change if the adoption of new technology is associated with a change in the economy's production structure. This is the tack that Goldin and Katz (1998) and Heckman, Lochner, and Taber (1998) take. For instance, suppose that the aggregate production function is

$$y = [\theta u^\rho + (1 - \theta)s^\rho]^{\alpha/\rho} k^{(1-\alpha)}. \tag{17}$$

Now, a change in k will not affect the skill premium, $(\partial y/\partial s)/(\partial y/\partial u)$, other things equal. But imagine that a new technology, say computers or electricity, comes along that favors skilled relative to unskilled labor. Heckman, Lochner, and Taber (1998) operationalize this by assuming that the production function shifts in such a way that θ drifts downwards.[15] This raises the skill premium. Note that equation (17) is an *aggregate* production function. Therefore, a decrease in θ affects new and old capital alike, and investment in new capital is *not* necessary to implement the technological progress.

A production structure that shifts toward skilled labor can easily extend to the case in which investment in new capital *is* required to implement new technologies. Suppose, as Solow (1960) does, that technological progress applies only to new capital goods, and write

$$y_v = A_v[\theta_v u_v^\rho + (1 - \theta_v)s_v^\rho]^{\alpha/\rho} k_v^{(1-\alpha)},$$

where y_v, u_v, s_v, and k_v are the output and inputs of the vintage "v" technology, and θ_v is a parameter of the production that is specific to that technology. The newer vintage technologies are better, and so A_v is increasing in v. At each date, there will, in general, be a range of v's in use, especially if there is some irreversibility in the capital stock. Now suppose that θ_v is decreasing in v. That is, better technologies require less unskilled labor. The adoption of such technologies will raise the skill premium. In this type of model, the skill premium rises because of technological adoption and not directly because of a rise in the stock of capital.

Caselli (1997) suggests, instead, that each new technology demands its own types of skills, skills that may be easier or harder to acquire, relative to the skills required by older technologies. If the skills associated with a new technology are relatively hard to learn and if people's abilities to learn

15. They estimate that $[(1 - \theta)/\theta]$ has grown at a rate of 3.6 percent since the 1970s. This yields roughly the right magnitude of the increase in the college-high school wage gap.

differ, a technological revolution may raise income inequality by rewarding those able enough to work with the new technology.

Matching Workers and Machines

Fixed proportions between workers and machines. The previous arguments presume that workers differ in skill, or in their ability to acquire it. A basic implication of the vintage capital model is that a range of vintages of machines will be in use at any date. Can one somehow turn this implication into a proposition that workers, too, will be different? If a worker could operate a continuum of technologies and if he could work with infinitesimal amounts of each of a continuum of machines of different vintages, the answer would be *no*, because each worker could operate the "market portfolio" of machines. As soon as one puts a finite limit to the number of machines that a worker can simultaneously operate, however, the model generates inequality of workers' incomes. To simplify, assume that the worker can operate just one machine at a time and, moreover, that each machine requires just one worker to operate it. In other words, there are fixed proportions between machines and workers. Under these assumptions, inequality in workers' skills will emerge because of differential incentives for people to accumulate skills, and it translates into a nondegenerate distribution of skills. The following is an outline of the argument.

1. *Production function.* Suppose that one machine matches with one worker. The output of the match is given by the constant-returns-to-scale production function $y = F(k, s)$, where k is the efficiency level of the machine, and s is the skill level supplied by the worker. Machine efficiency and skill are complements in that $\partial^2F/\partial k\partial s > 0$.
2. *Growth of skills.* Let v be the fraction of his or her time that the worker spends working, and let h denote the level of his or her human capital. Then $s = vh$. Suppose that the worker can invest in raising h as follows: $dh/dt = \eta(1 - v)h$, where $1 - v$ is the fraction of his or her time spent learning.
3. *Growth of machine quality.* New machines, in turn, also get better. In other words, there is investment-specific technological progress. Suppose that anyone can produce a new machine of quality k according to the linearly homogenous cost function $C(k, \mathbf{k})$, where $\mathbf{k}$ is the average economy-wide quality of a newly produced machine.
4. *Balanced growth.* This setup produces a balanced growth path with some interesting features, as Jovanovic (1998) details. First, it results in nondegenerate distributions of machine efficiency and of worker skill. This can be true even if everybody was identical initially. It occurs because the scarcity of resources means that it is not optimal to give everyone the latest machine. The distributions over capital and skills move rightward over time. Second, because $\partial^2F/\partial k\partial s > 0$, better workers match with the better

machines according to an assignment rule of the form $s = \Phi(k)$, with $\Phi' > 0$. Third, faster-growing economies should have a greater range over machine quality and skills.

6.5.2 Nelson and Phelps (1966) and Skill in Adoption

The previous subsection was based on the notion that skilled labor is better at *using* a new technology; the alternative view is that skilled labor is more efficient at *adopting* a technology and learning it. The original Nelson and Phelps (1966) formulation, and its subsequent extensions like Benhabib and Spiegel (1994), do not invoke the vintage-capital model. It will be invoked now.

Evidence on Adoption Costs and Their Interaction with Skill

When a new technology is adopted, output tends to be below normal while the new technology is learned. Indeed, output will often fall below that which was attained under the previous technology. In other words, the adoption of a new technology may carry a large foregone output cost incurred during the learning period. There is evidence that the use of skilled labor facilitates this adoption process.

1. Management scientists have found that the opening of a plant is followed by a temporary increase in the use of engineers whose job is to get the production process "up to speed" (Adler and Clark 1991).
2. Bartel and Lichtenberg (1987) provide evidence for the joint hypothesis that (a) educated workers have a comparative advantage in implementing new technologies, and (b) the demand for educated versus less educated workers declines as experience is gained with a new technology.
3. In a more recent study of 450 U.S. manufacturing industries from 1960 to 1990, Caselli (1999) finds that the higher an industry's nonproduction-production worker ratio was before 1975 (his measure of initial skill intensity), the larger was the increase in its capital-labor ratio over 1975 to 1990 period (a measure of the adoption of new capital goods).

Modelling the Role of Skill in Adoption

To implement the idea that skill facilitates the adoption process, let

$$y_\tau = z_\tau k_\tau^\alpha u_\tau^\beta$$

be the production function for the age τ technology, and k_τ and u_τ represent the amounts of capital and unskilled labor. Assume that the improvement in a plant's practice, $dz_\tau/d\tau$, depends upon the amount of skilled labor, s_τ, hired:

$$\frac{dz_\tau}{d\tau} = \vartheta(1 - z_\tau)s_\tau^\phi - \mu z_\tau.$$

There is an upper bound on the level of productivity that can be achieved with any particular vintage of capital. As the amount of unrealized potential $(1 - z_\tau)$ shrinks, it becomes increasingly difficult to effect an improvement. The initial condition for z, or its starting value as of when the plant is operational, is assumed to be inversely related to the rate of technological progress, g_q, in the following way:

$$z_0 = \psi g_q^{-\xi},$$

where ψ and ξ are positive parameters.

Such a formulation can explain the recent rise in the skill premium; the details are in Greenwood and Yorukoglu (1997). Suppose that in 1974 the rate of investment-specific technological progress rose, perhaps associated with the development of information technologies. This would have led to an increase in the demand for skill needed to bring the new technologies on line. The skill premium would then have risen, other things being equal.

6.6 Three Models of Endogenous Investment-Specific Technological Progress

It is simple to endogenize investment-specific technological progress. How? Three illustrations based on three different engines of growth will show the way:

1. Learning by doing, as in Arrow (1962).
2. Research in the capital goods sector à la Krusell (1998).
3. Human capital investment in the capital goods sector following Parente (1994).

6.6.1 Solow (1960) Meets Arrow (1962): Learning by Doing as an Engine of Growth

Arrow (1962) assumes that technological progress stems exclusively from learning by doing in the capital goods sector. There are no learning curves or diffusion lags in the sector that produces final output. In the capital goods sector, there are no direct costs of improving production efficiency. Instead, a capital goods producer's efficiency depends on cumulative aggregate output of the entire capital goods sector—or, what is the same thing, cumulative aggregate investment by the *users* of capital goods. Because each producer has a negligible effect on the aggregate output of capital goods, learning is purely external. The job of casting Arrow's notion of learning by doing in terms of Solow's vintage capital framework will now start.

Production of final goods and accumulation of capital. Population is constant; write the aggregate production function for final goods in per capita terms as $c + i = k^\alpha$, where c, i, and k are all per capita values, an innocuous normalization if returns to scale are constant. Physical capital accumulates as follows:

$$\frac{dk}{dt} = iq - \delta k. \tag{18}$$

Once again, q is the state of technology in the capital goods sector: Anyone can make q units of capital goods from a unit of consumption goods.

Learning by capital goods producers. Suppose that at date t, q is described by

$$q(t) = \nu\left[\int_0^\infty q(t - s)\mathbf{i}(t - s)ds\right]^{1-\alpha}, \tag{19}$$

where $\mathbf{i}(t - s)$ denotes the level of industry-wide investment at date $t - s$ in consumption units, and $q(t - s)\mathbf{i}(t - s)$ is the number of machine efficiency units produced at $t - s$. In equation (19), as in Arrow's model, the productivity of the capital goods sector depends on economy-wide cumulative investment.[16]

Let λ be the mass of identical agents in this economy—the economy's "size" or "scale." Then, in equilibrium, $\mathbf{i} = \lambda i$, so that equation (19) becomes

$$q(t) = \nu\lambda^{1-\alpha}\left[\int_0^\infty q(t - s)i(t - s)ds\right]^{1-\alpha}. \tag{20}$$

Endogenous balanced growth. Assume that consumers' tastes are described by

$$\int_0^\infty e^{-\rho t}\ln c(t)dt. \tag{21}$$

Let g_x denote the growth rate of variable x in balanced growth. The production function implies that because population is constant, $g_y = \alpha g_k$. Along a balanced growth path, output of the capital goods sector, or $q\lambda i$, grows at rate g_k so that equation (20) implies that $g_q = (1 - \alpha)g_k$. Thus, the price of capital goods, $1/q$, falls as output grows.

16. In order to simplify things, Arrow's (1962) assumption that there are fixed, vintage-specific proportions between machines and workers in production is dropped. This assumption can lead to the scrapping of capital before the end of its physical life span. (In his analysis, capital goods face sudden death at the end of their physical life span, unless they are scrapped first, as opposed to the gradual depreciation assumed here.) Also, Arrow assumes that machine producers' efficiency is an isoelastic function of the cumulative number of machines produced, whereas here it is assumed to be an isoelastic function of the cumulative number of *efficiency units* produced.

LEMMA 1. *If a balanced growth path exists, g_k satisfies the equation*

$$(22)\qquad \underbrace{\rho + \delta + g_k}_{\textit{Interest Rate}} = \underbrace{\alpha\nu\lambda^{1-\alpha}\left(1 + \frac{\delta}{g_k}\right)^{1-\alpha}}_{q\times MP_k}.$$

PROOF. First, from equation (18), $g_k = -\delta + qi/k$. Since g_k, and hence qi/k, must be constant, $g_k = g_q + g_i = g_q + g_y$, where the second equality follows from assuming that consumption and investment are constant fractions of income along the balanced growth path so that $g_i = g_c = g_y$. Second, consider the first-order condition of optimality for k. A forgone unit of consumption can purchase q units of capital that can rent for $\alpha k^{\alpha-1}q$. This must cover the interest cost, $\rho + g_y$, the cost of depreciation, δ, and the capital loss g_q, because capital goods prices are falling. This gives the efficiency condition $\alpha k^{\alpha-1} = (\rho + \delta + g_y + g_q)/q = (\rho + \delta + g_k)/q$. Third, in balanced growth, $q(t - s)i(t - s) = e^{-(g_q + g_i)s}q(t)i(t) = e^{-g_k s}\, q(t)i(t)$. Then, using equation (20), $q = \nu\lambda^{1-\alpha}(qi/g_k)^{1-\alpha}$, which yields $qk^{\alpha-1} = \nu\lambda^{1-\alpha}[(qi/k)/g_k]^{1-\alpha}$. Substituting the fact that $g_k = -\delta + (qi/k)$ into this expression gives $qk^{\alpha-1} = \nu\lambda^{1-\alpha}[(g_k + \delta)/g_k]^{1-\alpha} = \nu\lambda^{1-\alpha}(1 + \delta/g_k)^{1-\alpha}$. Recalling that $\alpha qk^{\alpha-1} = (\rho + \delta + g_k)$ yields equation (22). Q.E.D.

COROLLARY 2. *There exists a unique and positive solution to equation (22).*

PROOF. The left-hand side of equation (22) is positively sloped in g_k, with intercept $\rho + \delta$. The right-hand side is negatively sloped, approaching infinity as g_k approaches zero, and approaching $\alpha\nu\lambda^{1-\alpha}$ as g_k approaches infinity. Therefore, exactly one solution exists, and it is strictly positive. Q.E.D.

PROPOSITION 3. *Scale effect: A larger economy, as measured by λ, grows faster.*

PROOF. Anything that raises (lowers) the right-hand side of equation (22) raises (lowers) g_k. Anything that raises (lowers) the left-hand side of equation (22) lowers (raises) g_k.[17] Q.E.D.

Example 1. Set capital's share of income at 30 percent, the rate of time preference at 4 percent, and the depreciation rate at 10 percent. Hence, $\alpha = 0.3$, $\rho = 0.04$, and $\delta = 0.10$. Now, values are backed out for the parameters ν and λ that will imply the existence of an equilibrium in which capital goods prices fall at 4 percent a year; that is, an equilibrium with $g_q = 0.04$. This leads to the capital stock's growing at rate $g_k = 0.04/(1 -$

17. It follows immediately that g_k is increasing in ν, and decreasing in ρ.

0.3) = 0.057. To get this value of g_k to solve equation (22), it must transpire that ν and λ are such that $\nu\lambda^{1-\alpha} = 0.32$.

Applying the model to information technology. The pace of technological progress in information technologies has been nothing short of incredible. Consider the cost of processing, storing, and transmitting information. Jonscher (1994) calculates that between 1950 and 1980 the cost of one MIP (millions of instructions per second) fell at a rate of somewhere between 27 and 50 percent per year. Likewise, the cost of storing information dropped at a rate of somewhere between 25 and 30 percent per year from 1960 to 1985. Last, the cost of transmitting information declined at a rate somewhere between 15 and 20 percent per year over the period 1974 to 1994.

Why such a precipitous fall in the cost of information technology? Arrow's model gives a precise answer. Information technology is a general purpose technology, usable in many industries. The scale of demand for the capital goods embodying it, and hence the cumulative output of these capital goods, has therefore been large, and this may well have led to a faster pace of learning and cost reduction.

A more specialized technology such as, say, new coal-mining machinery, would be specific to a sector (coal mining) and would, as a result, be demanded on a smaller scale. Its cumulative output and investment would be smaller, and so would its learning-induced productivity gains. In terms of the model, the value of λ for information technologies exceeds the value of λ for coal mining equipment. This amounts to a scale effect on growth. A higher λ hastens the decline in capital goods prices, a fact that proposition (3) demonstrates.

6.6.2 Solow (1960) Meets Krusell (1997): Research as an Engine of Growth

In Krusell's model, the improvement in capital goods comes about through research.

Final goods producers. The production function for final goods, y, is

$$y = l^{1-\alpha}\int_0^1 k_j^{\alpha}dj, \tag{23}$$

where l is the amount of labor employed in the final output (or consumption) sector, and k_j is the employment of capital of type j. The consumption sector is competitive and rents its capital from capital goods producers each period.

Capital accumulation. Each type of capital, j, is produced and owned by a monopolist who rents out his stock of machines, k_j, on a period-by-period basis to users in the consumption goods sector. Technological progress occurs at the intensive margin; k_j grows as follows:

(24) $$\frac{dk_j}{dt} = -\delta k_j + q_j x_j,$$

where x_j is spending by capital goods producer j, measured in consumption units, and q_j represents the number of type j machines that a unit of consumption goods can produce. In other words, q_j is the production efficiency of monopolist j.

Research by capital goods producers. Capital goods producer j can raise q_j by doing research. Because the markup the producer charges is proportional to q_j, he or she has the incentive to undertake research in order to raise q_j. If the producer hires h_j workers to do research, then monopolist j can raise his or her efficiency as follows:

(25) $$\frac{dq_j}{dt} = q_j^{\gamma}\mathbf{q}^{1-\gamma}R(h_j),$$

where $R(\cdot)$ is an increasing, concave function. The term $\mathbf{q} = \int_0^1 q_j dj$ is the average level of productivity across all sectors, and γ is an index of the product-specific returns to R&D. This term affects incentives to do research (if $\gamma = 0$, no incentive exists), but it does not affect the growth accounting procedure as long as h_j lends itself to measurement.

Symmetric equilibrium. Consider a balanced growth path where each monopolist is a facsimile of another so that $k_j = k(t)$, $q_j = q(t)$, and $h_j = h(t)$, etc. The first three equations become

(26) $$y = l^{1-\alpha}k^{\alpha},$$

(27) $$\frac{dk}{dt} = --\delta k + qx,$$

and

(28) $$\left(\frac{1}{q}\right)\left(\frac{dq}{dt}\right) = R(h).$$

Then the capital stock can be represented as

(29) $$k(t) = \int_{-\infty}^{t} e^{-\delta(t-s)}q(s)x(s)ds.$$

Equation (27) is of the same form as equation (4) of section 6.4, and the evolution of q now has a specific interpretation: Investment-specific technological progress is driven by research. Note from equation (27) that all new investment, $x(t)$, is in the frontier technology in the sense that it embodies $q(t)$ efficiency units of productive power per unit of consumption foregone.

Difficulties with research-based models. Although it captures features that section 6.2 argued were essential for understanding the U.S. growth experience, there are three problems with Krusell's model.

1. *A predicted secular increase in the growth rate.* Equation (28) implies that the rate of growth in the United States should have risen over time because in the U.S. data, and for that matter in most economies, h has trended upwards. Jones (1995) discusses the incongruity of these implications of research-based models with evidence.
2. *A positive scale effect.* To see this, take two identical economies and merge them into a single one that has twice as much labor and capital as the individual economies did. Now, hold the types of capital producers constant, because adding more types is tantamount to inventing new capital goods. If each agent behaves as previously described, then initially $y = 2l^{1-\alpha}k^{\alpha}$. Additionally, each firm could now use twice as much research labor so that q and k would grow faster. Alternatively, in this hypothetical experiment one could instead assume (realistically so perhaps) that the merged economy would have not a monopoly but a duopoly in each machine market. The consequences of such an assumption are not entirely clear, however, because the old allocation of labor to research would still not be an equilibrium allocation in the new economy. Competition in the machine market would lead to lower profits for the producers of machines, and this would reduce their incentives to do research and reduce growth. This would partially offset, and even reverse, the positive effect of scale on growth. These arguments make clear, moreover, that the scale problem in this model has nothing whatsoever to do with spillovers in research. The arguments go through intact even if $\gamma = 1$. The scale effect works through the impact that a larger product market has on firms' incentives to improve their efficiency.
3. *The resources devoted to research are small.* Most nations report no resources devoted to research, and only 3 percent or so of U.S. output officially goes to R&D. Because so much technology, even in the United States, is imported from other countries, research-based models make more sense at the level of the world than they do at the national level.

6.6.3 Solow (1960) Meets Parente (1994): Vintage Human Capital as an Engine of Growth

Parente (1994) offers a vintage human capital model without physical capital. This section adds a capital goods sector to his model. Once again, endogenous technological progress occurs in the capital goods sector only.

As in the Arrow and Krusell models, this technological progress is then passed onto consumption goods producers in the form of a beneficial "pecuniary external effect"—the falling relative price of capital.

Imagine an economy with two sectors of production: consumption and capital goods. The consumption goods sector is competitive and enjoys no technological progress. The productivity growth occurring in this sector arises because its capital input becomes less expensive over time relative to consumption goods and relative to labor.

The capital goods sector is competitive too, and its efficiency rises over time. A capital goods producer can, at any time, raise the grade of his technology, in the style of Zeckhauser (1968) and Parente (1994), but at a cost. The producer has an associated level of expertise at operating his grade of technology. This increases over time due to learning by doing. The profits earned by capital goods producers are rebated back each period to a representative consumer (who has tastes described by equation (21) and supplies one unit of labor).

Consumption goods sector. The production function for consumption goods is

$$c = k^{\alpha} l^{1-\alpha}, \tag{30}$$

where k and l are the inputs of capital and labor. This technology is unchanging over time.

Capital goods sector. Capital goods are homogeneous, but the technology for producing them can change at the discretion of the capital goods producer. A capital goods producer's technology is described by $o = Azh^{1-\alpha}$, where o is the producer's output of capital goods, A denotes the grade of the technology the producer is using, z represents the producer's level of expertise, and h is the amount of labor the producer employs. The price of capital is represented by p and the wage by w, both in consumption units. At any date the producer's labor-allocation problem is static and gives rise to flow profits given by

$$\max_h (pAzh^{1-\alpha} - wh) = \alpha\left[\frac{(1-\alpha)}{w}\right]^{(1-\alpha)/\alpha} (pAz)^{1/\alpha} \equiv \pi(A, z, p, w).$$

Learning by doing. Suppose that a producer's expertise on a given technological grade, A, grows with experience in accordance with

$$\frac{dz}{d\tau} = \lambda(1 - z), \qquad \text{for } 0 \le z \le 1,$$

where τ is the amount of time that has passed since the producer adopted the technology. Observe that while $z < 1$, the producer learns by doing. In contrast to Arrow's assumptions, this rate does not depend on the volume of output, however, but simply on the passage of time. Eventually, the producer learns everything and $z \to 1$, which is the maximal level of expertise.[18]

Let z_τ represent the accumulated expertise of a producer with τ years of experience. With an initial condition $z_0 = \tilde{z} < 1$, the above differential equation has the solution

$$z_\tau = 1 - (1 - \tilde{z})e^{-\lambda\tau} \equiv Z_\tau(\tilde{z}) \tag{31}$$

for $\tau \geq 0$.

Upgrading. A capital goods producer can, at any time, upgrade the technology he or she uses. If the producer switches from using technology A to A' he or she incurs a switching cost of $\kappa + (\vartheta A')/A$, measured in terms of lost expertise. The idea is that the bigger the leap in technology the producer takes, the less expertise he can carry over into the new situation. Observe that

1. There is no exogenously specified technological frontier here. That is, A' is unconstrained,[19] and yet producers do not opt for an A' that is as large as possible.[20]
2. In sharp contrast to Arrow (and to Krusell, unless his $\gamma = 1$), there is no technological spillover in human capital accumulation across producers.

Figure 6.6 plots the evolution of TFP for a producer.

Balanced growth. The balanced growth path will be uncovered through a guess-and-verify procedure. To this end, suppose that the economy is in balanced growth at date zero. It seems reasonable to conjecture that con-

18. The functional form of the learning curve is taken from Parente (1994). Zeckhauser (1968) considers a wider class of learning curves.

19. One caveat on Parente's model: The choice of A' is constrained by the fact that the level expertise following an adoption z_0 cannot be negative. This constraint could be removed by choosing a different form for the loss of expertise caused by upgrading. An example of a functional form that would accomplish this is $z_0 = (A'/A)^\vartheta (z_T - \kappa)$, where $z_T > \kappa$ is the level of expertise just before the adoption and $\vartheta < 0$.

20. Chari and Hopenhayn (1991) and Jovanovic and Nyarko (1996) also focus on human capital–based absorption costs. These models provide a microfoundation for why switching costs should be larger when the new technology is more advanced. This implies that when a firm does switch to a new technology, it may well opt for a technology that is inside the frontier. This implication separates the human capital vintage models from their physical capital counterparts, because the latter all imply that all new investment is in frontier methods.

Search frictions can also lead firms to adopt methods inside the technological frontier. In the models of Jovanovic and Rob (1989) and Jovanovic and MacDonald (1994), it generally does not pay for firms to invest time and resources to locate the best technology to imitate.

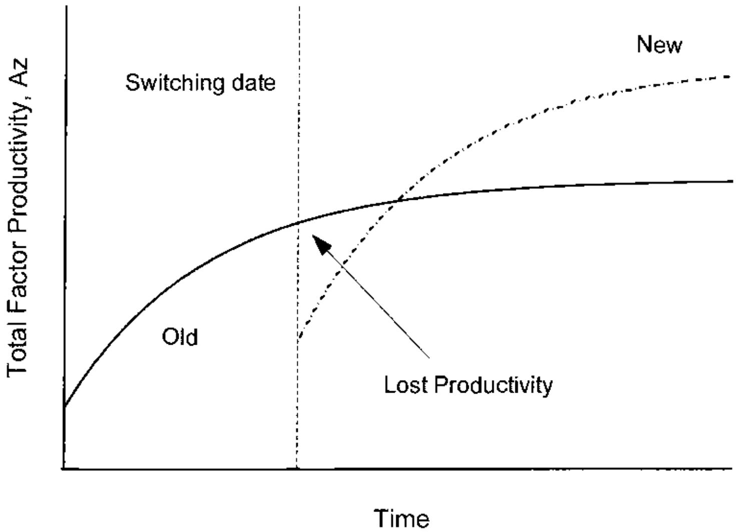

Fig. 6.6 Evolution of productivity

sumption, investment, aggregate output, and the stock of capital will all grow at constant rates, denoted as before by g_c, g_i, g_y, and g_k. If consumption and investment are to remain a constant fraction of income, then $g_c = g_i = g_y$. From equation (30), $g_y = \alpha g_k$.

Properties of the conjectured steady-state growth path.

1. Each capital goods producer will choose to upgrade A after interval T and by a factor ξ. Neither T nor ξ depends on time. Define g_A by $\xi = e^{g_A T}$. Then $g_A = (1/T) \ln \xi$ is the average growth rate of each producer's A.
2. At a point in time the age of the technologies in use are uniformly distributed over the interval $[0, T]$, with $1/T$ producers using each type of technology.
3. All producers using a technology of given grade have the same level of expertise.
4. z_0 solves the equation $z_0 = Z_T(z_0) - \kappa - \vartheta\xi$.

By properties 1 and 2, the distribution of technologies will be shifting continually to the right over time, and the maximal technological grade in use at time t, or $A_0(t)$, will grow exponentially: $A_0(t) = A_0(0)e^{g_A t}$. Let A_τ denote the level of technology that was upgraded τ periods ago. Then from the viewpoint of the producer that is using it, τ is the age of the technology A_τ. Then properties 1 and 2 also imply that $A_\tau(t) = A_0(t)e^{-g_A \tau}$. In a steady state, $(1/A_\tau[t])(dA_\tau[t]/dt) = g_A$ for all τ. The normalization $A_0(0) = 1$ will be employed in what follows.

By properties 3 and 4, $Z_\tau(z_0)$ is the level of expertise of each plant that uses the technology A_τ. By properties 2 and 3, the output of capital goods is

$$\left(\frac{A_0}{T}\right)\int_0^T e^{-g_A\tau} Z_\tau(z_0) h_\tau^{1-\alpha} d\tau = \frac{i}{p}, \tag{32}$$

where i is aggregate investment measured in consumption units and p is the price of capital in terms of consumption. The left-hand side of equation (32) implies that the output of capital goods grows at the rate $g_k = g_A$, given that h_τ is constant over time (a fact demonstrated below). In growth-rate form, equation (32) then reads $g_p = g_i - g_k$. If investment is to remain a constant fraction of income, $g_i = g_y$ must hold. Therefore, $g_p = g_y - g_k = -(1 - \alpha)g_A$.

It is easy to establish that distribution of labor remains constant across grades. That is, $h_\tau(t)$ will not depend on t. Optimal labor hiring in the consumption goods sector implies that $(1 - \alpha)(k/l)^\alpha = w$. If wages grow at the same rate as output, $g_y = \alpha g_k$, then l will be constant over time. Likewise, a capital goods producer using an age τ technology will hire labor according to the condition $(1 - \alpha)h_\tau^{-\alpha} = w/(pA_\tau z_\tau)$. Because $g_p = -(1 - \alpha)g_A$ and $g_A = g_k$, the right-hand side of this expression is constant over time, and, therefore, so is h_τ.

Producer's problem. In balanced growth, prices and wages grow at constant rates as a function of t, which therefore plays the role of the "aggregate state." To a capital goods producer, the state variables are his expertise, z, and his technological grade, A. Hence the Bellman equation pertaining to his decision problem is

$$V(A, z; t) = \max_{T', A'} \left\{ \int_t^{t+T'} \Pi[A, Z_{s-t}(z); s] e^{-r(s-t)} ds + e^{-rT'} V\left(A', Z_{T'}(z) - \kappa - \vartheta \frac{A'}{A}; t + T'\right) \right\},$$

where $\Pi(A, Z_{s-t}(z); s) \equiv \pi[A, Z_{s-t}(z), p(s), w(s)]$. The interest rate r is presumed to be constant.

A stationary (s, S) upgrading policy. One still needs to verify that the balanced growth equilibrium has the property, conjectured in (1), that capital goods producers choose to upgrade A by a constant factor ξ, and after a constant waiting time, T. If so, then there exists an (s, S) policy on the interval $[z_0, z_T]$ so that z always starts from $z_0 \geq 0$ (just after a technological upgrade) and increases up to the point $z_T \leq 1$, which triggers the next upgrade, a return of z to z_0, and so on. To show that the balanced growth path is of this form, the following property of the profit function is helpful.

LEMMA 4. *Let* $a(t) \equiv A/A_0(t)$. For $s \geq t$, $\Pi(A, z; s) = e^{\alpha g_A t}\Phi[a(t), z, s - t]$.

PROOF. Because $g_p = -(1 - \alpha)g_A$ and $g_w = \alpha g_A$, it follows that profits for period s, $\Pi(A, z; s)$, can be expressed as $[\alpha(1 - \alpha)/w(s)]^{(1-\alpha)/\alpha}[p(s)Az]^{1/\alpha} = e^{(1/\alpha)g_A t}a(t)^{(1/\alpha)}z^{(1/\alpha)} \times c(0)e^{-[(1-\alpha)/\alpha+1-\alpha]g_A s}$, where $c(0)$ is a constant whose value depends on some time-0 variables. Next, using the fact that $(1/\alpha) - [(1 - \alpha)/\alpha + 1 - \alpha] = \alpha$ allows the statement $\Pi(A, z; s) = e^{\alpha g_A t}a(t)^{(1/\alpha)}z^{(1/\alpha)} \times c(0)e^{-[(1-\alpha)/\alpha+1-\alpha]g_A(s-t)}$ to be written. Finally, the claim follows by setting $\Phi[a(t), z, s - t] = a(t)^{(1/\alpha)}z^{(1/\alpha)} \times c(0)e^{-[(1-\alpha)/\alpha+1-\alpha]g_A(s-t)}$. Q.E.D.

Let $a' = A'/A_0(t + T')$. Then because $A_0(t) = e^{g_A t}$ and $A'/A = [a'/a][A_0(t + T')/A_0(t)] = e^{g_A T'}(a'/a)$, the Bellman equation becomes

$$V(ae^{g_A t}, z; t) = e^{\alpha g_A t}\max_{T',a'}\Bigg[\int_t^{t+T'}\Phi(a, Z_{s-t}(z), s - t)e^{-r(s-t)}ds + e^{-rT'-\alpha g_A t}V\Big(a'e^{g_A(t+T')}, Z_{T'}(z) - \kappa - \vartheta e^{g_A T'}\frac{a'}{a}; t + T'\Big)\Bigg].$$

Now observe that after a change of variable $x = s - t$, $\int_t^{t+T'}\Phi(a, Z_{s-t}(z), s - t)e^{-r(s-t)}ds = \int_0^{T'}\Phi(a, Z_x(z), x)e^{-rx}dx$. Then, if one writes $B(a, z; t) \equiv e^{-\alpha g_A t}V(ae^{g_A t}, z, t)$, the Bellman equation becomes

$$B(a, z; t) = \max_{T',a'}\Bigg[\int_0^{T'}\Phi(a, Z_x(z), x)e^{-rx}dx + e^{-(r-\alpha g_A)T'}B\Big(a', Z_{T'}(z) - \kappa - \vartheta e^{g_A T'}\frac{a'}{a}; t + T'\Big)\Bigg]. \tag{1'}$$

PROPOSITION 5. *The upgrading policy is stationary.*

PROOF. Consider equation (1′). For $\kappa > 0$, one can bound the optimal policy T' away from zero. Now, because αg_A is the growth of consumption, optimal savings behavior by consumers implies that $r - \alpha g_A > 0$.[21] Therefore, the operator is a contraction, and by starting an iteration with a function B that does not depend on t, one finds that the unique fixed point, $B(a, z)$ does not depend on t. Denote the optimal decision rules by $T'(a, z)$ and $a'(a, z)$. Since T' and a' do not depend on t, upgrading by each producer will be periodic and by the same multiple. This is all conditional on the existence of a balanced growth path. Q.E.D.

Definition of balanced growth. For a balanced growth path to exist there must be a triple (ξ, T, z_0) such that, for all t,

$$T'(1, z_0) = T, \tag{33}$$

21. Suppose that tastes are described by equation (21). Then, along a balanced growth path, $r = \rho + g_y = \rho + \alpha g_k$. Hence, $r - \alpha g_k = \rho$.

(34) $$a'(1, z_0) = 1,$$

and

(35) $$z_0 = Z_T(z_0) - \kappa - \vartheta\xi.$$

In this case, output, consumption, and investment grow at the rate

$$\alpha g_A = \left(\frac{\alpha}{T}\right)\ln\xi.$$

Together, equations (33)–(35) imply that an economy that starts on the steady-state growth path described by properties 1–4, given earlier in section 6.6.3, remains on it. Equations (33) and (34) pertain to the optimal behavior of a producer right after he has upgraded his technology. Right after an upgrade, the producer has a technology $A = A_0(t)$ and hence $a = 1$. He must then choose to wait T periods (equation [33]) and, at that point, he must choose to upgrade A by a constant factor ξ (equation [34]). Finally, given the T and ξ that he has chosen, his expertise must be the same after each upgrade (equation [35]).

The changeover process. The above model generates a balanced growth path along which income grows and the relative price of capital falls. Technological progress in the capital goods sector is endogenous. At each point in time there is a distribution of capital goods producers, using a variety of production techniques. Each capital goods producer decides when to upgrade his technology. Because there is a cost of doing so, in terms of loss of expertise, he will economize on the frequency of doing this. In the real world such adoption costs may be quite high, implying that the changeover process will be slow.[22]

Salter (1966) noted some time ago that the changeover process at the plant level is slow. He quotes Hicks as stating that an "entrepreneur by investing in fixed capital equipment gives hostages to fortune. So long as the plant is in existence, the possibility of economizing by changing the method or scale of production is small; but as the plant comes to be renewed it will be in his interests to make a radical change" (4). The above model captures this process, but here the capital investment is in knowledge.

As evidence of the slow changeover process, consider table 6.1, compiled by Salter (1966). The first column presents labor productivity for plants using the best-practice or the most up-to-date techniques at the time. Average labor productivity across all plants is reported in the second column. As Salter (1966) notes,

22. This type of model may have some interesting transitional dynamics. Imagine starting off with some distribution of technologies where producers are bunched up around some particular technique. What economic forces will come to bear to encourage them not to all upgrade around the same date in the future? How long will it take for the distribution to smooth out?

Table 6.1 Best and Average Practice in the U.S. Blast Furnace Industry (tons of pig iron per man-hour, 1911–26)

Year	Best-Practice Plants	Industry Average
1911	0.313	0.140
1917	0.326	0.150
1919	0.328	0.140
1921	0.428	0.178
1923	0.462	0.213
1925	0.512	0.285
1926	0.573	0.296

Source: Salter (1966).

> In this industry, average labor productivity is only approximately half best-practice productivity. If all plants were up to best-practice standards known and in use, labor productivity would have doubled immediately. In fact, a decade and a half elapsed before this occurred, and in the meantime the potential provided by best-practice productivity had more than doubled. (6)

Salter's (1966) findings have weathered time well, for, in a recent study of plants' TFP in 21 four-digit textile industries, Dwyer (1998) finds that average TFP among the second (from top) decile divided by the average TFP among the ninth-decile plants (a procedure that is relatively insensitive to outliers) falls between 2 and 3.

6.7 Conclusions: Solow (1956, 1957) versus Solow (1960)

Forty years ago, Solow wrote some classic papers on economic growth. In the classic Solow (1956) paper, technological progress rained down from heaven. The invention of new techniques and their implementation were free. Technological progress affected the productivity of all factors of production, capital and labor, both new and old, alike. By contrast, in Solow (1960) technological advance was embodied in the form of new capital goods. Its implementation is not free because one must invest to realize the benefits from it. This form of technological advance is dubbed investment specific.

So, which framework is better? It is argued here that the Solow (1960) vintage capital model is. First, over the postwar period there has been tremendous technological advance in the production of new capital goods. The relative price of capital goods has declined at about 4 percent per year. Second, the variation in productivity across plants in the United States is tremendous. It is hard to believe that some of this is not due to differences in capital goods employed. In fact, Bahk and Gort (1993) have found that a one-year change in the average age of capital is associated with a 2.5 to 3.5 percent change in a plant's output. Now, there is evidence suggesting that the pace of investment-specific technological progress has picked up since the 1970s with the advent of information technologies. Supposing that this is true, variants of the Solow (1960) framework, modified to incorporate implementation costs and skilled labor, can go some way in explaining the recent productivity slowdown and the rise in wage inequality.

Why does the source of technological progress matter? It may have implications for economic growth, unemployment, or other issues that society cares about. For example, if technological progress is embodied in the form of new capital goods, then policies that reduce the costs of acquiring new equipment (such as investment tax credits for equipment buyers or R&D subsidies for equipment producers) may stimulate growth.[23]

Appendix

Data Definitions and Sources

The sample period is 1948–92, and all data are annual. Real income, y, is defined as nominal GDP minus nominal gross housing product deflated by the implicit price deflator for personal consumption expenditure on nondurables and nonhousing services. The GDP series were obtained from the Bureau of Economic Analysis (STAT-USA website), and the prices series were taken from CITIBASE. Real private sector nonresidential net capital stock, k, and its equipment and structures components (k_e and k_s, respectively), were again downloaded from the BEA. Total private sector hours employed, l, is obtained from CITIBASE (series name LHOURS). Labor share, $1 - \alpha$, was constructed by dividing nominal total compensation of employees by nominal income minus nominal proprietor's income. The data are again from the BEA website. The (standard) rate of technological progress is calculated by using

$$\ln z_t - \ln z_{t-1} = \ln\left(\frac{y_t}{l_t}\right) - \ln\left(\frac{y_{t-1}}{l_{t-1}}\right) - \frac{(\alpha_t + \alpha_{t-1})}{2}\left[\ln\left(\frac{k_t}{l_t}\right) - \ln\left(\frac{k_{t-1}}{l_{t-1}}\right)\right],$$

so that

$$z_t = \exp\left[\sum_{j=1949}^{t} (\ln z_j - \ln z_{j-1})\right],$$

with $z_{1948} = 1$.

The price index for producer's durable equipment is taken from Gordon (1990, until 1983, and Krusell et al. 2000 after 1983). The relative price of

23. Stimulating growth does not necessarily improve welfare. The sacrifice in terms of current consumption may be prohibitively high.

equipment, p, is calculated by deflating this price index by the consumer price index. Investment-specific technological progress, q, is then just equal to $1/p$.

To calculate the k_e series used in section 6.3, a discrete approximation to equation (6) is used. The starting point for the equipment series was taken to be the value for k_e implied by the model's balanced-growth path for the year 1947 as taken from the relationship

$$k_e = \frac{qi_e}{(g_y + 1)(g_q + 1) - (1 - \delta_e)},$$

where i_e is nominal gross private domestic fixed investment in producer's durable equipment (from the BEA website) deflated by the price index for personal consumption expenditure on nondurables and non-housing services.

The Mismeasurement of Neutral Technological Progress in Traditional Growth Accounting

To simplify, assume that the labor force is constant. Now, suppose that a growth accountant failed to incorporate investment-specific technological progress into his analysis. He would construct his capital stock series according to

$$\text{(A1)} \qquad \frac{d\tilde{k}}{dt} = i - \delta\tilde{k}.$$

This corresponds to measuring the stock of capital at historical cost in output units. Using equation (2), he would obtain the following series describing neutral technological progress:

$$g_{\tilde{z}} = g_y - \alpha g_{\tilde{k}}.$$

Because by assumption all growth in output must derive from growth in the capital stock, it must transpire from equation (3) that $g_y = \alpha g_k$ so that

$$g_{\tilde{z}} = \alpha(g_k - g_{\tilde{k}}).$$

Hence, any change in the measured Solow residual arises solely from mismeasurement in the capital stock. To gain an understanding of this equation, suppose that the economy was gliding along a balanced growth path. From equations (4) and (A1), it is clear that in this situation, $g_k - g_{\tilde{k}} = g_q$, implying $g_{\tilde{z}} = \alpha g_q$. Although the growth accountant may have killed off investment-specific technological progress in his misspecification of the law of motion of capital, it has resurrected itself in the form of neutral technological progress.

The model demands that GDP should be measured in consumption units, the numeraire. Doing so is important. What would happen if the growth accountant used standard real GDP numbers? Specifically, let GDP be measured as

$$\tilde{y} = c + \bar{p}qi,$$

where $\bar{p}$ is some base year price for capital goods. Applying equation (2), the growth accountant would obtain

$$\text{(A2)} \qquad g_{\tilde{z}} = g_{\tilde{y}} - \alpha g_k = g_{\tilde{y}} - g_y .$$

The difference in the growth rates between the traditional measure of GDP, $\tilde{y}$, and the consumption based one, y, will be picked up as neutral technological progress. Now,

$$g_{\tilde{y}} = \left(\frac{c}{\tilde{y}}\right)g_c + \bar{p}\left(\frac{iq}{\tilde{y}}\right)g_i + \bar{p}\left(\frac{iq}{\tilde{y}}\right)g_q .$$

Along a balanced growth path $g_c = g_i = g_y$. Because q is growing, it must therefore transpire that $(c/\tilde{y}) \rightarrow 0$ and $(\bar{p}iq/\tilde{y}) \rightarrow 1$. Hence, asymptotically

$$g_{\tilde{y}} = g_i + g_q = g_y + g_q ,$$

so that from equation (A2), $g_{\tilde{z}} = g_q$.

Once again, investment-specific technological progress has masqueraded itself as neutral technological progress.

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Comment Barry Bosworth

This paper is a very coherent survey of a set of recent growth models that have attempted to extend Solow's contributions to growth theory. The authors are able to integrate within the framework of a formal model many of the ideas that have been put forth as explanations for the post-1973 productivity slowdown. They also develop a very interesting model in which they stress the role of embodied technical change.

They identify three issues that they believe cannot be fully explained within Solow's original one-sector growth model (1956): (a) the post-1973 slowdown, (b) the falling relative price of capital, and (c) the rising relative wage premium for skilled labor. They argue that the first is due to the high costs of implementing the new technologies and the second to rapid technical growth in the capital goods producing industry, and that the increased skill premium is a reflection of greater complementarity between the new capital and skilled labor. All of these conclusions are plausible and have been argued in various forms by others; but we suffer from a shortage not of possible explanations, but of evidence on which explanations are true. Let me address the authors' explanations in reverse order.

Wage Dispersion

The authors are certainly right to point to the sharp widening of wage inequality as a dramatic new feature of economic change over the last quarter century, and we can agree that technology is part of the explanation for a change in the distribution of labor demand by skill level, but its linkage to the productivity slowdown may be more tenuous. Nearly all industrial countries have had a productivity slowdown, but only a few have experienced a widening of the wage distribution. At the international level, it is interesting to note that countries with high growth rates seem to have narrower wage distributions. In addition, the dispersion of U.S. wage rates has increased in more dimensions than skill, suggesting that it is not only a reflection of an increased skill bias in technical change. Finally, if the dominant change were computers and similar technologies, I would expect it to rebound to the advantage of the young, who, it is said, find it easier to adapt; yet the age profile seems to have become more steep, and the widening of the wage distribution is concentrated among the young.

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