On modelling the natural rate of unemployment with indivisible labour

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Abstract. This analysis investigates modelling the natural rate of unemployment in settings where labour's utilization has some lumpy aspect to it. Specifically, the introduction of various nonconvexities into tastes and technology lead to unemployment in general equilibrium. Equilibria can emerge where (1) those currently unemployed have higher probabilities of being unemployed in the future than those currently employed, (2) some agents are jobless while others work overtime, and (3) seniority rules seemingly arise whereby old workers cannot be laid off so long as either new workers are being hired or some workers are doing overtime. The relative welfare levels of employed and unemployed agents are analysed.

Sur la modelisation du taux naturel de chômage quand le travail est indivisible. Cette analyse examine un modèle du taux naturel de chômage dans des cas où le travail utilisé n'est pas divisible en unités fines. Spécifiquement, l'introduction de nonconvexités variées dans les goûts et la technologie entraîne du chômage en équilibre général. Des équilibres peuvent se réaliser pour lesquels [i] ceux qui sont en chômage pour le moment ont une probabilité plus grande d'être employés dans l'avenir que ceux qui sont employés pour le moment, [ii] certains agents économiques sont sans emploi alors que d'autres font du temps supplémentaire, [iii] des règles d'ancienneté se font supposément jour qui impliquent que de vieux employés ne peuvent pas être mis à pied tant que, soit de nouveaux employés sont engagés, soit certains travailleurs font du temps supplémentaire. On analyse aussi les niveaux relatifs de bien-être de ceux qui sont employés et de ceux qui ne le sont pas.

INTRODUCTION

For about the last fifteen years, economists have been concerned with the micro-economic foundations of employment theory. Friedman (1968) defined

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the concept of the natural rate of unemployment as 'the level that would be ground out by the Walrasian system of general equilibrium equations.' Generating unemployment in Walrasian frameworks has proved to be difficult to date. While such paradigms can easily provide a determination of equilibrium employment, making the jump to modelling equilibrium unemployment has proved to be elusive. Introducing 'frictions' to labour market trade would seem to be an obvious place to start when one is trying to formulate theories of *equilibrium unemployment*. This is more difficult than it seems, however, and many attempts along these lines have had limited success in this regard.

As a case in point, consider the implicit labour contracting model in environments with asymmetric information as formulated by Azariadis (1983), Grossman and Hart (1983), and others – see Hart (1983) for a survey of this literature. Here workers contract with firms for a wage / employment package designed to stabilize labour income in the face of fluctuations in economic activity. Since only firms observe the underlying shocks affecting the economy, workers will believe entrepreneurs' declarations that a 'bad' state-of-the-world has occurred only if, at the time of wage cuts, an observable such as employment is also sufficiently reduced. This necessity for contracts to be written in a manner ensuring that entrepreneurs truthfully report the state-of-the-world *can* result in the 'underemployment' of labour relative to a world with symmetric information, in the sense that average hours per worker is lower.[1] However, it does not result in any agents being unemployed. Thus, while such contracting can lead to real wage rigidity and underemployment, it does not *in and of itself* result in unemployment.

A notable exception which does bridge the hiatus between modelling employment and unemployment in Walrasian frameworks is the seminal equilibrium search model developed by Lucas and Prescott (1974). Here, an aggregate economy made up of many individual markets subject to idiosyncratic disturbances is constructed. Given the stochastic structure of the economy, a worker is continually undertaking calculations to decide whether or not it is in his best interest to quit his current job in a particular industry and enter a search process in pursuit of a higher return to work effort elsewhere. The model generates a natural rate of unemployment together with equilibrium distributions of wages, employment, and unemployment across industries.

An alternative approach to modelling unemployment in Walrasian frameworks has recently been advanced by Rogerson (1985). Here the gap between the notions of equilibrium employment and unemployment is bridged by introducing indivisibilities into economic agents' labour supply decisions. Such non-convexities turn out to be capable of generating unemployment within the context of equilibrium models. Rogerson also shows how these indivisibilities

1 A standard result in implicit labour contracting theory for environments with symmetric information is that all employment decisions are undertaken efficiently in the standard Walrasian manner – see Hart (1983).

can be handled by a simple extension to the standard competitive equilibrium construct. The fact that simple modifications of the standard competitive equilibrium model can be applied to model what many may view as intrinsically 'non-market clearing' phenomena, has recently been stressed by Prescott and Townsend (1984).

Drawing on Rogerson's work, Hansen (1985) simulates a stochastic growth model with indivisible labour. He finds that this type of model mimics quite well certain U.S. labour market stylized facts, such as the large fluctuations in aggregate hours worked relative to average productivity. Greenwood and Huffman (1987) borrow from Rogerson to construct a dynamic equilibrium model which is capable of explaining, in a theoretical sense, the covariance properties between unemployment and inflation – or Phillips curve correlations – both conditioned and unconditioned upon exogenous factors such as the current growth rate of the money supply, the level of productivity, etc.

A closer examination of modelling the natural rate of unemployment with indivisible labour is undertaken here. The analysis focuses on how different ways of introducing non-convexities into the specification of taste and technology can produce various observed labour market phenomena. For instance, it is shown that this approach can generate appealing serial correlation properties in agents' employment histories. Specifically, those currently working can have higher (lower) probabilities of being (un)employed in the future than those currently not working. Also, it is demonstrated that the introduction of such non-convexities into the economic environment allows for a simultaneous determination of *both* the total number of individuals working in the economy and the amount of hours worked per employed agent. This permits an explanation of certain observed facts such as some labour force participants working overtime while others are unemployed. Additionally, seniority rights for workers can seemingly emerge in such environments. In particular, situations exist where old workers can never be laid off so long as new workers are being hired or some agents are working overtime. Finally, some discussion of the determinants of the relative consumption and welfare levels of the employed versus unemployed is undertaken.

The remainder of the paper is organized as follows. The second section contains a general description of the economic environment to be employed. In the third section the representative agent's optimization problem is cast. The link between non-convexities and non-separable preferences on the one hand, and the serial correlation properties of agents' employment histories on the other, is studied in the fourth section. The relationship between employed and unemployed agents' consumption and welfare levels is examined in the subsequent section. In the sixth section it is shown how the introduction of non-convexities into the economy's technology can allow for desirable properties in agents' employment histories as well as a determination of both the extensive and intensive margins of labour force participation. Conclusions are offered in the final section.

THE ECONOMIC ENVIRONMENT

Consider the following model of a two-period economy inhabited by a continuum of identical agents distributed uniformly over the interval [0, 1]. An agent's goal in life is to maximize the expected value of his lifetime utility as given by

$$E_0[\mathrm{U}(c_1, c_2, l_1, l_2)],$$

where c_t and l_t denote his period-t consumption and work effort (for $t = 1, 2$). The function $\mathrm{U}(\cdot)$ is assumed to be strictly concave and continuously twice differentiable. It is strictly increasing in its first two arguments and strictly decreasing in its last two. Furthermore, it is assumed that

$$\lim_{c \to 0} \mathrm{U}_1(c, \cdot, \cdot, \cdot) = \infty, \text{ and } \lim_{c \to 0} \mathrm{U}_2(\cdot, c, \cdot, \cdot) = \infty.$$

Aggregate output in period t is described by the constant-return-to-scale production process

$$y_t = f(L_t, K) \qquad t = 1, 2$$

with L_t and K representing the aggregate amount of labour (measured in efficiency units) and capital employed in this period. The function $f(\cdot)$ is strictly increasing in both its arguments and twice continuously differentiable with

$$\lim_{L \to 0} f_1(L, K) = \infty, \lim_{L \to l} f_1(L, K) = 0, \qquad \text{and } f(0, K) = 0.$$

The ownership of capital is completely diversified with each agent possessing K units of capital.

Each agent is assumed to face a non-convexity in his labour supply decision. In particular, in each period t he either works the amount $l_t \geqq l$, or he does not work at all. There is also a non-convexity connected with firms' hiring decisions. Specifically, a firm must incur a once-and-for-all cost γ associated with employing a worker. It has long been recognized that the Pareto optimality property of a competitive equilibrium is not necessarily destroyed by the presence of non-convexities in either agents' tastes or firms' production technologies – see Rothenberg (1960). As is shown by Rogerson (1985), however, an extension of agents' choice sets to allow for the possibility of lotteries over consumption and labour allocations, will in general improve individuals' (ex ante) welfare in the situation currently under consideration. Specifically, imagine the production process as being owned by a competitive firm who offers an individual an income-employment contract of the following form: In the first period the firm and the agent agree on the probability ϕ_1^w that the individual will be called in to work l units. An individual who is chosen to be employed receives a wage-cum-dividend payment from the firm designed to allow him to undertake c_1^w units of consumption. The probability of being unemployed in the first period is $\phi_1^u = (1 - \phi_1^w)$, and in this state the individual receives income sufficient to provide for c_1^u units of consumption. Since all agents are identical ex ante and uniformly distributed over the unit interval, equilibrium first-period aggregate employment is given by $L_1 = \phi_1^w l$. Associated with this level of employment in the first period, firms in the aggregate are incurring hiring costs in the amount $\gamma\phi_1^w$.

For the second period, the probability of an agent's being called in to work is assumed to depend upon his previous employment history. Thus, the firm and the worker agree on the probability $\phi_2^w(w)$ that the individual will be called in to work the amount $l_2(w) \geqq l$ in the second period *conditional* on the fact that he was *employed* in the first period, and on the probability $\phi_2^w(u)$ that he will work the amount $l_2(u) \geqq l$ in the second period *conditional* on the fact that he was unemployed in the previous period. Hence, more formally, $\phi_2^w(w) = \text{prob}\,(l_2(w) \geqq l \mid l_1 \geqq l)$ and $\phi_2^w(u) = \text{prob}\,(l_2(u) \geqq l \mid l_1 = 0)$. It immediately follows that $(1 - \phi_2^w(w))$ and $(1 - \phi_2^w(u))$ represent the probabilities of being unemployed in the second period conditional on whether the agent was working or not, respectively, during the first period. The unconditional probabilities of being employed, ϕ_2^w, and unemployed, ϕ_2^u, in the second period are given by $\phi_2^w = \phi_1^w\phi_2^w(w) + \phi_1^u\phi_2^w(u)$, and $\phi_2^u = \phi_1^w\phi_2^u(w) + \phi_1^u\phi_2^u(u)$. In the second period the firm makes the individual a wage-cum-dividend payment designed to provide for *one* of four consumption possibilities, viz. $c_2^w(w)$, $c_2^u(w)$, $c_2^w(u)$, and $c_2^u(u)$, depending upon *both* the worker's current and past employment status. For example, if the individual is currently working but was unemployed last period, the firm pays him income sufficient to allow for consumption in the amount, $c_2^w(u)$. The unconditional probability associated with this consumption possibility is $(1 - \phi_1^w)\phi_2^w(u)$. It may be the case that an individual's labour productivity is affected by his work experience. Specifically, let agents who worked in the first period be more productive in the second period by a factor of $\lambda \geqq 1$ over those who don't have work experience. Second-period aggregate employment (in efficiency units) is then given by $L_2 = [\phi_1^w\phi_2^w(w)\lambda l_2(w) + (1 - \phi_1^w)\phi_2^w(u)l_2(u)]$. Finally, firms in the second period are incurring hiring costs in the amount $\gamma(1 - \phi_1^w)\phi_2^w(u)$, which are associated with hiring previously unemployed agents.

THE REPRESENTATIVE AGENT'S OPTIMIZATION PROBLEM

The decision-making of consumer-workers and firms in competitive equilibrium can be summarized by the outcome of the following 'representative' agent / firm's programming problem with the choice variables being c_1^w, c_1^u, $c_2^w(w)$, $c_2^w(u)$, $c_2^u(w)$, $c_2^u(u)$, $\phi_1^w = (1 - \phi_1^u)$, $\phi_2^w(w) = (1 - \phi_2^u(w))$, $\phi_2^w(u) = (1 - \phi_2^u(u))$, l_1, $l_2(w)$, and $l_2(u)$:

$$\text{Max } E_0[\mathrm{U}(\cdot)] = \phi_1^w E[\mathrm{U}(\cdot) \mid l_1 \geqq l] + (1 - \phi_1^w)E[\mathrm{U}(\cdot) \mid l_1 = 0]$$
$$= \phi_1^w[\phi_2^w(w)\mathrm{U}(c_1^w, c_2^w(w), l_1, l_2(w))$$

$$+ (1 - \phi_2^{w}(w))\mathbf{U}(c_1^{w}, c_2^{u}(w), l_1, 0)]$$
$$+ (1 - \phi_1^{w})[\phi_2^{w}(u)\mathbf{U}(c_1^{u}, c_2^{w}(u), 0, l_2(u))$$
$$+ (1 - \phi_2^{w}(u))\mathbf{U}(c_1^{u}, c_2^{u}(u), 0, 0)] \quad (1)$$

s.t.

$$\phi_1^{w}c_1^{w} + (1 - \phi_1^{w})c_1^{u} = f(\phi_1^{w}l, K) - \gamma\phi_1^{w} \quad (2)$$

$$\phi_1^{w}[\phi_2^{w}(w)c_2^{w}(w) + (1 - \phi_2^{w}(w))c_2^{u}(w)] + (1 - \phi_1^{w})[\phi_2^{w}(u)c_2^{w}(u) + (1 - \phi_2^{w}(u))c_2^{u}(u)] = f(\phi_1^{w}\phi_2^{w}(w)\lambda l_2(w) + (1 - \phi_1^{w})\phi_2^{w}(u)l_2(u), K) - \gamma(1 - \phi_1^{w})\phi_2^{w}(u) \quad (3)$$

$$0 \leqq \phi_2^{w}(w) \leqq 1 \quad (4)$$

$$0 \leqq \phi_2^{w}(u) \leqq 1 \quad (5)$$

$$l_1 \geqq l \quad (6)$$

$$l_2(w) \geqq l \quad (7)$$

$$l_2(u) \geqq l. \quad (8)$$

The expected value of the agent's lifetime utility function, or the maximand, is given by equation (1). The economy's resource constraints for the first and second periods are described by equations (2) and (3). The next two equations bind the conditional probabilities $\phi_2^{w}(w)$ and $\phi_2^{w}(u)$ to have values in the unit interval. (Unlike the unconditional probabilities, there do not seem to be any natural restrictions on tastes and technology to ensure this outcome). Finally, equations (6), (7), and (8) restrict employed agents to supply at least l units of labour effort.

The solution to the above choice problem is completely summarized by the following set of efficiency conditions – in addition to equations (2) and (3)

$$E[\mathbf{U}_1(\cdot) \mid l_1 \geqq l] = E[\mathbf{U}_1(\cdot) \mid l_1 = 0] \qquad (\text{for } c_1^{w}, c_1^{u}) \quad (9)$$

$$\mathbf{U}_2(c_1^{w}, c_2^{w}(w), l_1, l_2(w)) = \mathbf{U}_2(c_1^{w}, c_2^{u}(w), l_1, 0)$$
$$= \mathbf{U}_2(c_1^{u}, c_2^{w}(u), 0, l_2(u))$$
$$= \mathbf{U}_2(c_1^{u}, c_2^{u}(u), 0, 0) \qquad (\text{for } c_2^{i}(j),\ i, j = u, w) \quad (10)$$

$$E[\mathbf{U}(\cdot) \mid l_1 = 0] - E[\mathbf{U}(\cdot) \mid l_1 \geqq l]$$
$$= E[\mathbf{U}_1(\cdot) \mid l_1 \geqq l][f_1(L_1, K)l - \gamma - (c_1^{w} - c_1^{u})]$$
$$+ \mathbf{U}_2(\cdot)\{f_1(L_2, K)[\phi_2^{w}(w)\lambda l_2(w) - \phi_2^{w}(u)l_2(u)]$$
$$- (E[c_2 \mid l_1 \geqq l] - E[c_2 \mid l_1 = 0])\} \qquad (\text{for } \phi_1^{w}) \quad (11)$$

$$\mathbf{U}(c_1^{w}, c_2^{u}(w), l_1, 0) - \mathbf{U}(c_1^{w}, c_2^{w}(w), l_1, l_2(w))$$

$$+ \psi_2^w/\phi_1^w \geqq \mathrm{U}_2(\cdot)[f_1(L_2, K)\lambda l_2(w) - (c_2^w(w) - c_2^u(w))]$$
$$\text{(with equality if } \phi_2^w(w) > 0) \qquad (12)$$

$$\mathrm{U}(c_1^u, c_2^u(u), 0, 0) - \mathrm{U}(c_1^u, c_2^w(u), 0, l_2(u)) + \psi_2^u/(1 - \phi_1^w)$$
$$\geqq \mathrm{U}_2(\cdot)[f_1(L_2, K)l_2(u) - \gamma - (c_2^w(u) - c_2^u(u))]$$
$$\text{(with equality if } \phi_2^w(u) > 0] \qquad (13)$$

$$-E[\mathrm{U}_3(\cdot) \mid l_1 \geqq l] \geqq E[\mathrm{U}_1(\cdot)]f_1(L_1, K)$$
$$\text{(with equality if } l_1 > l) \qquad (14)$$

$$-\mathrm{U}_4(c_1^w, c_2^w(w), l_1, l_2(w)) \geqq \mathrm{U}_2(\cdot)\lambda f_1(L_2, K)$$
$$\text{(with equality if } l_2(w) > l) \qquad (15)$$

$$-\mathrm{U}_4(c_1^u, c_2^w(u), 0, l_2(u)) \geqq \mathrm{U}_2(\cdot)f_1(L_2, K)$$
$$\text{(with equality if } l_2(u) > l) \qquad (16)$$

$$\psi_2^w[1 - \phi_2^w(w)] = 0 \qquad (17)$$

$$\psi_2^u[1 - \phi_2^w(u)] = 0, \qquad (18)$$

where ψ_2^w and ψ_2^u are the Lagrange multipliers associated with the constraints (4) and (5). Note that equations (12), (13), (14), (15), and (16) hold strictly as $\phi_2^w(w)$, $\phi_2^w(u)$, $l_1 - l$, $l_2(w) - l$, and $l_2(u) - l$ are greater than zero, respectively.

By glancing at the above set of equations, it may appear that not much can be gleaned from the simple problem posed. It turns out that the structure of taste and technology plays a crucial role in determining the nature of the model's solution. By specifying taste and technology in various ways, the above system of equations can be used to gain insight on two issues: first, the serial correlation properties of agents' states of employment and, second, the relative welfare levels of employed and unemployed workers.

NON-SEPARABLE PREFERENCES AND SERIALLY CORRELATED EMPLOYMENT

It will now be investigated whether the above model is capable of generating serial correlation in agents' employment histories. More precisely, the question to be addressed is whether the combination of non-convexities and non-separabilities in preferences can lead to equilibria arising where currently employed agents have higher probabilities of working in the future than those currently employed. That is, can the model generate $\phi_2^w(w) > \phi_2^w(u)$ as a solution? In pursuit of this end, the utility function $\mathrm{U}(\cdot)$ will be specialized to

$$\mathrm{U}(\cdot) = U(c_1) + \beta U(c_2) + V(l_1, l_2),$$

with period-t labour effort, l_t, being restricted to be an element of the two-point non-convex set $\{0, l\}$. Thus, $U(\cdot):R_+ \times R_+ \times \{0, l\} \times \{0, l\} \to R$. The advantage of this specification is that an agent's consumption in each period is independent of his employment history, a fact evident from (9) and (10), so that $c_1^w = c_1^u = c_1$, and $c_2^i(j) = c_2$ for $i, j = w, u$. The concavity and separability properties of the momentary utility function, $U(\cdot)$, imply that agents desire to smooth (perfectly) consumption in a given period across their employment characteristics. Finally, to abstract away from technological considerations let $\gamma = 0$ and $\lambda = 1$.

A complete characterization of the model's solution in the current situation – that is, a determination of c_1, c_2, ϕ_1^w, $\phi_2^w(w)$, $\phi_2^u(w)$ – is given by the following analogues to (2), (3), (11), (12), and (13):

$$c_1 = f(\phi_1^w l, K) \tag{19}$$

$$c_2 = f(\phi_1^w \phi_2^w(w)l + (1 - \phi_1^w)\phi_2^w(u)l, K) \tag{20}$$

$$E[V(\cdot) \mid l_1 = 0] - E[V(\cdot) \mid l_1 = l] = U'(c_1)f_1(\phi_1^w l, K)l + \beta U'(c_2)f_1(\phi_1^w\phi_2^w(w)l + (1 - \phi_1^w)\phi_2^w(u)l, K) \times [\phi_2^w(w) - \phi_2^w(u)]l \tag{21}$$

$$[V(l, 0) - V(l, l)] + \psi_2^w/\phi_1^w \geqq \beta U'(c_2)f_1(\phi_1^w\phi_2^w(w)l + (1 - \phi_1^w)\phi_2^w(u)l, K)l \text{ (with equality if } \phi_2^w(w) > 0) \tag{22}$$

$$[V(0, 0) - V(0, l)] + \psi_2^u/(1 - \phi_1^w) \geqq \beta U'(c_2)f_1(\phi_1^w\phi_2^w(w)l + (1 - \phi_1^w)\phi_2^w(u)l, K)l \text{ (with equality if } \phi_2^w(u) > 0). \tag{23}$$

(The complementary slackness conditions (17) and (18) governing ψ_2^w and ψ_2^u also hold here.)

An examination of the above set of equations will now be conducted in order to learn more about the serial correlation properties of an agent's state of employment generated by the model. In particular, many would hold the view that a person's probability of being employed (unemployed) in this period increases (decreases) if the individual were employed (unemployed) in the last period. Thus, the focus of the investigation will be on developing the set of necessary and sufficient conditions which allow for $0 \leqq \phi_2^w(u) < \phi_2^w(w) \leqq 1$ to emerge as a solution to the model. (Note that such a solution implies that $0 \leqq \phi_2^u(w) < \phi_2^u(u) \leqq 1$.) An analysis of equations (22) and (23) is central to the issue being pursued. The right-hand sides of these expressions illustrate the marginal benefits of having a larger fraction of the labour force working. The left-hand sides (ignoring the ψ_2 terms) represent the expected utility losses associated with a rise in the probability of being called into work. An important observation to make is that the right-hand sides of these two conditions are identical. Thus, not surprisingly, in the current situation the marginal benefit from either increasing the employment rate of those who were previously employed or those who were unemployed is the same.

PROPOSITION 1. *Suppose that* $[V(l, 0) - V(l, l)] \neq [V(0, 0) - V(0, l)]$. *Then the conditional probabilities,* $\phi_2^w(w)$ *and* $\phi_2^w(u)$, *have values determined in the following manner:*

$$0 \leqq \phi_2^w(u) < \phi_2^w(w) \leqq 1 \tag{24}$$

if and only if

$$[V(l, 0) - V(l, l)] < [V(0, 0) - V(0, l)], \tag{25}$$

and

$$0 \leqq \phi_2^w(w) < \phi_2^w(u) \leqq 1 \tag{26}$$

if and only if

$$[V(0, 0) - V(0, l)] < [V(l, 0) - V(l, l)]. \tag{27}$$

Proof. Observe that $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 1$ cannot be a solution to the model, since this situation is precluded by the assumption that $\lim_{\phi^w \to 1} f_1(\phi^w l, K) = 0$, which results in a violation of the necessary conditions (22) and (23). Likewise, a solution of the form $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 0$ cannot transpire, since this event is ruled out by the assumption that $f(0, K) = 0$ and $\lim_{c \to 0} U'(c) = \infty$, which again would contradict (22) and (23). The postulated relationship between (24) and (25) will now be demonstrated, with the proof of that between (26) and (27) being entirely analogous.

(Necessity). Suppose that (24) holds. Then equation (22) is binding, because $\phi_2^w(w) > 0$. Also, since $\phi_2^w(u) < 1$, it happens that $\psi_2^u = 0$ (from (18)). Therefore, since the right-hand sides of (22) and (23) are equal, $[V(l, 0) - V(l, l)] + \psi_2^w/\phi_1^w \leqq [V(0, 0) - V(0, l)]$, which yields the desired result as $\psi_2^w/\phi_1^w \geqq 0$.

(Sufficiency). Alternatively, assume that (25) holds. This implies that (22) must hold with equality. To see this, suppose that it didn't hold. Then (23) must hold with equality, because $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 0$ can't be a solution to the model. But then from (22) and (23) it follows that $[V(l, 0) - V(l, l)] > [V(0, 0) - V(0, l)] + \psi_2^u/(1 - \phi_1^w)$, which contradicts the initial assumption that (25) holds. Now, first, if (23) is also binding, then from (22), (23), and (25) it transpires that $\psi_2^w/\phi_1^w - \psi_2^u/(1 - \phi_1^w) = [V(0, 0) - V(0, l)] - [V(l, 0) - V(l, l)] > 0$. Since both ψ_2^w and ψ_2^u can't be strictly positive – for this would imply $\phi_2^w = \phi_2^w(w) = \phi_2^w(u) = 1$ – it follows that here $0 \leqq \phi_2^w(u) < \phi_2^w(w) = 1$. Second, if (23) is slack, then $0 = \phi_2^w(u) < \phi_2^w(w) \leqq 1$. These two inequalities yield (24). □

The next natural question to ask is under what conditions is $[V(l, 0) - V(l, l)] \lesseqgtr [V(0, 0) - V(0, l)]$? This question is answered by the following proposition.

PROPOSITION 2. *Suppose that $V_{12}(\cdot)$ is either* **strictly** *positive or negative for* **all** *values of l_1, $l_2 \in [0, l]$. Then*

$$[V(l, 0) - V(l, l)] \lesseqgtr [V(0, 0) - V(0, l)]$$

if and only if

$$V_{12}(\cdot) \gtreqless 0.$$

Proof. By the Fundamental Theorem of Calculus, the first statement is equivalent to

$$\int_0^l V_2(l, x)dx \gtreqless \int_0^l V_2(0, x)dx,$$

which occurs as

$$V_{21}(\cdot) \gtreqless 0. \square$$

Thus, the situation described in statement (24) will happen when the sign of $V_{21}(\cdot)$ is positive, while that characterized by (26) will arise when $V_{21}(\cdot)$ is negative. This makes intuitive sense. Consider the case where $V_{21}(\cdot) > 0$ which, as was just demonstrated, is a necessary condition for $0 \leqq \phi_2^w(u) < \phi_2^w(w) \leqq 1$. Here more (less) work in the first period lowers (increases) the marginal disutility of working in the second. This is equivalent to leisure in the two periods being net complements in the Edgeworth-Pareto sense. Not surprisingly then, for an optimal labour contract to draw mostly from the pool of individuals who were employed in the first period, it must happen that this set of agents has the lowest marginal disutility of working. In fact, it will be shown next that in this situation the firm will always *fully exhaust* the pool of individuals who were employed in the first period before it hires *any* agents who were unemployed then; that is, if $\phi_2^w(u) > 0$, then $\phi_2^w(w) = 1$. 'Old' workers seemingly have seniority in that they will never be laid off so long as 'new' workers are being hired.

PROPOSITION 3. *Suppose $V_{12}(\cdot)$ is either strictly positive or negative for all values of l_1, $l_2 \in [0, l]$. Then*

(a) *$\phi_2^w(w) < 1$ implies $\phi_2^w(u) = 0$*
if and only if
$V_{12}(\cdot) > 0$,

and

(b) *$\phi_2^w(u) < 1$ implies $\phi_2^w(w) = 0$*
if and only if
$V_{12}(\cdot) < 0$.

Proof. (a) Assume that $V_{12}(\cdot) > 0$ and $\phi_2^w(w) < 1$. Then from propositions 1 and 2 it is known that $0 \leqq \phi_2^w(u) < \phi_2^w(w) < 1$. This implies from (17) and

(18) that $\psi_2{}^u = \psi_2{}^w = 0$. Also, it must be the case that equation (22) is binding. Thus $\phi_2{}^w(u) = 0$, because equation (23) must be slack, since it has an identical right-hand side to (22) but a larger left-hand one by proposition 2 and $\psi_2{}^u = \psi_2{}^w = 0$. Conversely, suppose that $\phi_2{}^w(w) < 1$ and $\phi_2{}^w(u) = 0$. In this situation $0 = \phi_2{}^w(u) < \phi_2{}^w(w) < 1$, which necessitates by propositions 1 and 2 that $V_{12}(\cdot) > 0$.

(b) Proved by similar argument. □

A few words will now be said about the case where $V_{12}(\cdot)$ is negative; that is when leisure in adjacent periods are Edgeworth-Pareto substitutes. Now working in one period increases the marginal utility of leisure in the other. In this situation individuals are rotated through employment, with the firm in period two hiring first from the pool of period-one unemployed agents; that is, if $\phi_2{}^w(w) > 0$, then $\phi_2{}^w(u) = 1$. Thus, while the indivisibility prevents employment from being shared evenly at a point in time it is, so to speak, being shared across time. Note that this 'work-sharing' arrangement does not generate empirically appealing properties in individual agents' employment histories. To do this here would require introducing some form of non-convexities into the production technology. The issue of non-convexities in technology will be pursued later. Thus, if the combination of non-convexities and non-separabilities in preferences are to provide an explanation for observed labour market phenomena, then leisure in adjacent periods must be Edgeworth-Pareto complements, or $V_{12}(\cdot) > 0$. This will be a maintained hypothesis for the rest of this section.

Given the maintained hypothesis that $V_{12}(l_1, l_2) > 0$ *for all* $l_1, l_2 \in [0, l]$, it is known from propositions 1, 2, and 3 that the equilibrium solution for $\phi_2{}^w(u)$ and $\phi_2{}^w(w)$ must lie in one of two mutually exclusive sets as expressed below.

$$(\phi_2{}^w(u), \phi_2{}^w(w)) \in \{\{(x, y): x = 0, 0 < y \leqq 1\} \cup \{(x, y): 0 < x < y = 1\}\}. \quad (28)$$

Note that if $\phi_2{}^w(u)$ and $\phi_2{}^w(w)$ have values in the first set, then the second-period aggregate employment rate is less than or equal to the first-period one, since $\phi_2{}^w = \phi_1{}^w\phi_2{}^w(w) \leqq \phi_1{}^w$. Alternatively, if the values for $\phi_2{}^w(u)$ and $\phi_2{}^w(w)$ lie in the second set, then the aggregate employment rate in period two exceeds that in the first period, since $\phi_2{}^w = \phi_1{}^w + (1 - \phi_1{}^w)\phi_2{}^w(u) > \phi_1{}^w$. The feasibility of the latter possibility characterizing solutions to the model would appear to be related to the notion of time discounting. On the one hand, with a positive rate of time preference, agents, loosely speaking, prefer to consume goods today rather than tomorrow. On the other hand, they would rather work tomorrow than today. Obviously, aggregate consumption and employment are directly related to one another through the goods market clearing conditions ((19) and (20)). It might be expected intuitively that if the representative agent's rate of time preference for goods is higher than it is for leisure, then the *equilibrium* time profile for consumption will be downward sloping while the corresponding one for leisure will slope upward.

To investigate this conjecture more closely, let the representative consumer / worker's rate of time preference be restricted in the following manner

$$V_1(x, y)/V_2(y, x) \leqq 1/\beta \equiv 1 + \rho \qquad \text{for all } x, y \in [0, l]. \tag{29}$$

That is, the marginal disutility associated with working x units today (along with y tomorrow) is less than the marginal disutility of working x units tomorrow (and y today) multiplied by a factor of $1/\beta$. For *constant* labour effort and consumption profiles across time, the agent's (gross) rate of time preference for leisure is bounded above by his one for consumption, $1 + \rho$. Simply put, the individual's rate of impatience for consuming leisure cannot *exceed* that for goods. Clearly, this restriction holds for separable utility functions of the form $V(l_1, l_2) = W(l_1) + \beta W(l_2)$. The standard Cobb-Douglas utility function $V(l_1, l_2) = A(\bar{l} - l_1)^\alpha(\bar{l} - l_2)^\delta$, defined for $l_1, l_2 \in [0, l]$ where $l < \bar{l}$ and $\alpha > \delta$, also exhibits this property together with Edgeworth-Pareto complementarity if α and δ are chosen so that $(\alpha/\delta)[\bar{l}/(\bar{l} - l)]^{\alpha-\delta} < 1/\beta$.

PROPOSITION 4. *If $V_{12}(x, y) > 0$ and $V_1(x, y)/V_2(y, x) \leqq 1/\beta$ for all $x, y \in [0, l]$, then*

$$0 = \phi_2{}^w(u) < \phi_2{}^w(w) \leqq 1.$$

Proof. From (28) it suffices to show that equilibria where $(\phi_2{}^w(u), \phi_2{}^w(w)) \in \{(x, y) : 0 < x < y = 1\}$ cannot exist. Suppose to the contrary that they can. Then situations occur where $\phi_2{}^w > \phi_1{}^w$ so that $U'(f(\phi_2{}^w l, K))f_1(\phi_2{}^w l, K)l < U'(f(\phi_1{}^w l, K))f_1(\phi_1{}^w l, K)l$. For this possibility to be realized, however, it must happen that (from (21), and (23))[2]

$$[V(0, 0) - V(0, l)] < \beta[V(0, l) - V(l, l)]. \tag{30}$$

Now by the Fundamental Theorem of Calculus, the above condition occurs if and only if

$$\int_0^l [V_2(0, x) - \beta(V_1(x, l)/V_2(l, x))V_2(l, x)]dx > 0. \tag{31}$$

Next, note that the restriction (29) imposed on agent's rate of time preference for leisure implies that

$$[V_2(0, x) - V_2(l, x)] \geqq [V_2(0, x) - \beta(V_1(x, l)/V_2(l, x))V_2(l, x)]. \tag{32}$$

Since $V_{12}(\cdot) > 0$, then $V_2(0, x) < V_2(l, x)$, and, by (32), the left-hand side of (31) will be negative, which is the required contradiction. □

Observe that structural unemployment in its severest form has been generated

2 In more detail, from equation (23) it is known that $[V(0, 0) - V(0, l)] = \beta U'(f(\phi_2{}^w l, K))f_1(\phi_2{}^w l, K)l$. Using this information in (21) together with the fact that $\phi_2{}^w(w) = 1$ yields $[V(0, l) - V(l, l)] = U'(f(\phi_1{}^w l, K))f_1(\phi_1{}^w l, K)l$. The desired result follows immediately.

here. All those agents who were unemployed in the first period remain so in the second.

EMPLOYMENT AND WELFARE

An examination of the relationship between an individual's state of employment on the one hand and his levels of consumption and welfare on the other will now be undertaken. Clearly, in an ex ante sense the labour contract outlined in the third section improves agents' welfare, since it maximizes their expected utility. The issue of what happens to workers' welfare, ex post, once the state of their employment is known is unclear. Most would have the prejudice that the welfare levels of unemployed workers should be less than those of the employed. However, the model does not guarantee this outcome. In fact, in the previous section unemployed agents realize a higher welfare level than the employed. This result obtained because all individuals had the same level of consumption regardless of their state of employment. Since utility is a decreasing function in work effort, those who are employed are worse off than those who aren't. This feature is shared by the models of Rogerson (1985), Hansen (1985), and Greenwood and Huffman (1987).

The goal of this section is to discover whether unemployed workers can have lower levels of consumption and utility than employed ones in the model. A conjecture made here is that having a momentary utility function which is non-separable in consumption and labour effort is central to this issue. To investigate this hunch let the utility function, $\mathbf{U}(\cdot)$, be written as

$$\mathbf{U}(c_1, c_2, l_1, l_2) = U(c_1, l_1) + \beta U(c_2, l_2),$$

again being defined over the non-convex set $R_+ \times R_+ \times \{0, l\} \times \{0, l\}$. The virtue of the above form of an agent's lifetime utility function is that it is time separable. Furthermore, technological considerations will again be abstracted away by assuming that $\gamma = 0$ and $\lambda = 1$. These restrictions on the general form of the economic environment imply that an individual's second-period level of consumption and probability of being employed will be determined independently of his state of employment in the first period (see (10), (12) and (13)), so that formally $c_2^{\,i}(i) = c_2^{\,i}(j) \equiv c_2^{\,i}$, and $\phi_2^{\,i}(i) = \phi_2^{\,i}(j) = \phi_2^{\,i}$ for $i, j = w, u$. The model's solution for this restricted setting is given by the following analogue to the equation system described by (2), (3), and (9) to (13):

$$\phi_t^w c_t^w + (1 - \phi_t^w) c_t^u = f(\phi_t^w l, K) \qquad \forall\, t = 1, 2 \text{ (cf. (2) and (3))} \qquad (33)$$

$$U_1(c_t^w, l) = U_1(c_t^u, 0) \qquad \forall\, t = 1, 2 \text{ (cf. (9) and (10))} \qquad (34)$$

$$U(c_t^u, 0) - U(c_t^w, l) = U_1(\cdot\, t)[f_1(\phi_t^w l, K)l - (c_t^w - c_t^u)] \qquad \forall\, t = 1, 2 \text{ (cf. (11), (12), and (13))}. \qquad (35)$$

The key to learning about the relationship between consumption and employment within a period is given in the proposition presented below.

PROPOSITION 5. *Suppose that $U_{12}(c_t, l_t)$ is either strictly positive or negative for all values of $c_t \in R_+$ and $l_t \in [0, l]$. Then*

$$c_t^w \lesseqgtr c_t^u \text{ as } U_{12}(\cdot\ t) \lesseqgtr 0.$$

Proof. Define $c(l_t)$ by the equation $U_1(c(l_t), l_t) = \tilde{U}_1$. Thus

$$\left.\frac{dc_t}{dl_t}\right|_{U_1 = \tilde{U}_1} = -\frac{U_{12}(c(l_t), l_t)}{U_{11}(c(l_t), l_t)} \lesseqgtr 0 \text{ as } U_{12}(c(l_t), l_t) \lesseqgtr 0. \tag{36}$$

Now set $\tilde{U}_1 = U_1(c_t^u, 0)$, where c_t^u is the resulting solution from equations (33)–(35). The desired result then follows from the observation made below (cf. (34))

$$c_t^w = c_t^u + \int_0^l \left[\left.\frac{dc_t}{dl_t}\right|_{U_1 = \tilde{U}_1}\right] dl_t. \square$$

Suppose that consumption and work effort (leisure) are net complements (substitutes) in the Edgeworth-Pareto sense, so that $U_{12}(\cdot\ t) > 0$ – this will be a maintained hypothesis for the rest of this section. Here, working increases the marginal utility of consumption, which is not implausible, with the optimal labour contract therefore assigning a higher level of consumption to employed agents.

The next issue to address is whether or not employed workers have higher levels of welfare than unemployed ones. Observe from equation (35) that

$$U(c_t^w, l) \gtrless U(c_t^u, 0) \text{ as } f_1(\phi_t^w l, K)l - (c_t^w - c_t^u) \lesseqgtr 0. \tag{37}$$

Therefore, the employed can have a higher welfare level than the unemployed only when $c_t^w > c_t^u$, which necessitates that $U_{12}(\cdot\ t) > 0$. Now for employed agents actually to have a higher welfare level than unemployed ones, it must be the case that the higher level of consumption they receive more than compensates them for the loss in their leisure.

One would like to know whether this possibility can arise in the model. The answer seems to be a definite, but qualified, yes. As can be seen from (36), the amount of compensation an employed agent receives for his labour effort appears to be directly related to the magnitude of $-U_{12}(\cdot)/U_{11}(\cdot)$. There are reasonable limits, however, to how large this quantity can be made. In particular, if leisure is to remain a normal good, then the following restriction on this magnitude must hold: $-U_{12}(\cdot)/U_{11}(\cdot) < -U_2(\cdot)/U_1(\cdot)$ or equivalently $U_{11}(\cdot)[-U_2(\cdot)/U_1(\cdot)] + U_{12}(\cdot) < 0$.

It will now be investigated whether this factor is likely to be an important consideration when the welfare levels of employed and unemployed agents are compared.

PROPOSITION 6.

$$U(c_t^w, l) > U(c_t^u, 0)$$

only if

$$U_{11}(c_t, l_t)[-U_2(c_t, l_t)/U_1(c_t, l_t)] + U_{12}(c_t, l_t) > 0$$
$$\text{for some } c_t \in [c_t^u, c_t^w] \text{ and } l_t \in [0, l].$$

Proof. As in proposition 5, define the function $c(l_t)$ by the equation $U_1(c(l_t), l_t) = \tilde{U}_1$. Again set $\tilde{U}_1 = U_1(c_t^u, 0)$. Then $c_t^w = c(l)$ and $c_t^u = c(0)$, so that $U(c_t^w, l) = U(c(l), l)$ and $U(c_t^u, 0) = U(c(0), 0)$. Thus, by the Fundamental Theorem of Calculus

$$U(c_t^w, l) - U(c_t^u, 0) = \int_0^l [U_1(c(l_t), l_t) \left.\frac{dc_t}{dl_t}\right|_{U_1 = \tilde{U}_1} + U_2(c(l_t), l_t)]dl_t$$
$$= \int_0^l [-U_1(c(l_t), l_t)[U_{12}(c(l_t), l_t)/U_{11}(c(l_t), l_t)] + U_2(c(l_t), l_t)]dl_t \text{ (using (36))}.$$

Consequently, for $U(c_t^w, l) > U(c_t^u, 0)$ it must be the case that $-U_{11}(c_t, l_t) U_2(c_t, l_t)/U_1(c_t, l_t) + U_{12}(c_t, l_t) > 0$ for some $c_t \in [c_t^u, c_t^w]$ and $l_t \in [0, l]$. □

Thus, if employed individuals are to have a higher level of welfare than unemployed ones, the representative agent's utility function *must exhibit inferiority of leisure over some range* in the space $[c_t^u, c_t^w] \times [0, l]$. The above result does not seem to imply, however, that leisure has to be an inferior good for either employed or unemployed agents at their equilibrium levels of consumption and work effort.

Some further light can be shed upon the relationship between the welfare levels of employed and unemployed agents. In particular, Rogerson and Wright (1987) have extended the above results to show that employed agents are better (worse) off than unemployed ones if and only if an exogenous lump-sum increase in the economy's income causes the aggregate employment rate to rise (fall).

PROPOSITION 7. *(Rogerson and Wright, 1987): Let* $f(\phi_t^w l, K) = g(\phi_t^w l, K) + I_t$.

$$U(c_t^w, l) \lesseqgtr U(c_t^u, 0) \text{ as } d\phi_t^w/dI_t \lesseqgtr 0.$$

Proof. Displacing the system of equations (33), (34), and (35) with respect to I_t yields

$$d\phi_t^w/dI_t = U_{11}(c_t^w, l)U_{11}(c_t^u, 0)[U(c_t^w, l) - U(c_t^u, 0)]/\Omega_t$$

where

$$\Omega_t \equiv U_1(c_t^w, l)\{[U(c_t^w, l) - U(c_t^u, 0)]^2 U_{11}(c_t^w, l)U_{11}(c_t^u, 0)/U_1(c_t^w, l)^2$$

$$+ U_1(c_t^w, l)f_{11}(\phi_t^w l, K)[\phi_t^w U_{11}(c_t^w, l) + (1 - \phi_t^w)U_{11}(c_t^u, 0)]\} > 0. \square$$

Remark. It is immediately clear from propositions 6 and 7 that $d\phi_t^w/dI_t > 0$ only if $U_{11}(c_t, l_t)[-U_2(c_t, l_t)/U_1(c_t, l_t)] + U_{12}(c_t, l_t) > 0$ for some $(c_t, l_t) \in [c_t^u, c_t^w] \times [0, l]$.

Finally, another observation regarding the comparison between the welfare levels of those working and not working can be drawn. Recall from (37) that $U(c_t^w, l) \gtrless U(c_t^u, 0)$ as

$$f_1(\phi_t l, K)l - (c_t^w - c_t^u) \lesseqgtr 0.$$

Note by multiplying both sides of the above statement by ϕ_t^w and rearranging it can be equivalently expressed in the following manner:

$$c_t^u \lesseqgtr \phi_t^w c_t^w + (1 - \phi_t^w)c_t^u - f_1(\phi_t l, K)l\phi_t^w.$$

Next by utilizing (33) this can be rewritten in the form

$$c_t^u \lesseqgtr f(\phi_t^w l, K) - f_1(\phi_t^w l, K)l\phi_t^w,$$

which in turn can be further simplified through use of the constant-returns-to-scale assumption to obtain

$$c_t^u \lesseqgtr f_2(\phi_t^w l, K)K \equiv \pi_t.$$

Thus, in the situation where employed agents have a higher (lower) level of welfare, $U(c_t^w, l)$ than the unemployed, $U(c_t^u, 0)$, the latter are being taxed (subsidized) in the sense that their consumption, c_t^u, is less (greater) than their share of firms' profits, π_t – recall that the ownership of firms was assumed to be distributed uniformly across agents.

Ex ante, it is clearly beneficial for all agents to enter into the labour contract specified in the third section. Ex post, however, some subset of agents in the economy is made worse off in the sense that this collection of individuals is subsidizing the living standard of others. This draws into question whether the income transfers prescribed by the labour contract are likely to be enforceable.

MODIFICATIONS OF TECHNOLOGY

In the fourth and fifth sections it was shown that certain restrictions on preferences in conjunction with indivisible labour could help to produce behaviour of particular labour market variables which were consistent with casual observation. Some people may object to this line of inquiry, however, on the grounds that much of the behaviour of these labour market variables could be realistically attributable to the characteristics of technology rather than preferences. For instance, the fact that the probability of an employed agent's remaining employed is greater than the probability of an unemployed agent's attaining employment could be attributed to the higher productivity of agents with better employment histories, a technological consideration, rather than the form of his preferences. To these and other issues surrounding the specification of technology, attention will now be turned.

To begin with, consider the effect of introducing the non-convexities into the economic environment via lump-sum hiring costs. Specifically, let firms now incur once-and-for-all hiring costs, $\gamma > 0$, associated with employing a worker. In order to abstract from the types of considerations discussed in the previous two sections, preferences will be restricted to be of the inter- and intra-temporally separable form shown below:

$$\mathbf{U} = U(c_1) + V(l_1) + \beta[U(c_2) + V(l_2)], \tag{38}$$

with agents being free to choose the amount of effort they supply. Thus, $\mathbf{U}(c_1, c_2, l_1, l_2): R_+ \times R_+ \times R_+ \times R_+ \to R$. Finally, suppose labour productivity is unaffected by work experience; that is, let $\lambda = 1$. This assumption in conjunction with the separable form of preferences ensures that all agents who are employed in the second period work the same amount (see (15) and (16)) so that formally $l_2(w) = l_2(u) \equiv l_2$. Note that now firms and workers must decide upon the probabilities of working in each period ($\phi_1{}^w$, $\phi_2{}^w(w)$, $\phi_2{}^w(u)$) as well as the quantity of labour to be supplied by employed agents (l_1, l_2). The competitive equilibrium prevailing in this environment is characterized by the solution for (c_1, c_2, $\phi_1{}^w$, $\phi_2{}^w(w)$, $\phi_2{}^w(u)$, l_1, l_2) obtained from the following system of equations – in addition to the complementary slackness conditions (17) and (18).

$$c_t = f(\phi_t{}^w l_t, K) \qquad \text{(cf. (2) and (3))} \tag{39}$$

$$\begin{aligned} V(0) - V(l_1) &+ \beta\{E[V(\cdot\, 2)\,|l_1 = 0] - E[V(\cdot\, 2)\,|l_1 > 0]\} \\ &= U'(c_1)[f_1(\phi_1{}^w l_1, K)l_1 - \gamma] + \beta U'(c_2)\{f_1(\phi_2{}^w l_2, K)l_2[\phi_2{}^w(w) - \phi_2{}^w(u)] \\ &\qquad + \gamma\phi_2{}^w(u)\} \quad \text{(cf. (11))} \end{aligned} \tag{40}$$

$$\beta[V(0) - V(l_2)] + \psi_2{}^w/\phi_1{}^w \geqq \beta U'(c_2)f_1(\phi_2{}^w l_2, K)l_2 \quad \text{(cf. (12), with equality if } \phi_2{}^w(w) > 0) \tag{41}$$

$$\beta[V(0) - V(l_2)] + \psi_2{}^u/(1 - \phi_1{}^w) \geqq \beta U'(c_2)[f_1(\phi_2{}^w l_2, K)l_2 - \gamma] \quad \text{(cf. (13), with equality if } \phi_2{}^w(u) > 0) \tag{42}$$

$$-V'(l_t) = U'(c_t)f_1(\phi_t{}^w l_t, K) \qquad \text{(cf. (14), (15), and (16))} \tag{43}$$

(Recall $\phi_2{}^w = \phi_1{}^w\phi_1{}^w(w) + (1 - \phi_1{}^w)\phi_2{}^w(u)$).

Note that in general the above system of equations determines *both* the fraction of agents employed and hours worked per employed agent for both periods. In general, however, not much can be said about the properties of the solution. One fact is fairly immediate, however: individuals who were employed in the first period have seniority in the second period in the sense that they will

TABLE 1

γ	ϕ_1^w	l_1	c_1	$\phi_2^w(w)$	$\phi_2^w(u)$	l_2	c_2
0.45	1.00	0.84	0.44	1.00	*a*	0.90	0.66
0.50	0.90	0.90	0.41	1.00	1.00	0.92	0.63
0.55	0.77	0.97	0.39	1.00	1.00	0.96	0.60
0.60	0.69	1.03	0.375	1.00	0.67	1.03	0.56
0.65	0.62	1.09	0.36	1.00	0.50	1.11	0.53
0.70	0.56	1.15	0.35	1.00	0.40	1.18	0.50

a Value not relevant here

never be unemployed if there are *any* new recruits working. In other words, the firm will hire to work in the second period individuals who were previously unemployed only when the pool of first-period employed agents has been totally exhausted. This occurs because there is a lump-sum hiring cost attached to the former but not to the latter set of agents.

PROPOSITION 8. *$\phi_2^w(u) > 0$ only if $\phi_2^w(w) = 1$.*

Proof. Suppose $\phi_2^w(u) > 0$. Then from (42), $\beta[V(0) - V(l_2)] + \psi_2^u/(1 - \phi_1^w) = \beta U'(c_2)[f_1(\phi_2^w l_2, K)l_2 - \gamma]$. Using this fact in (41), generates $-\beta U'(c_2)\gamma - \psi_2^u/(1 - \phi_1^w) + \psi_2^w/\phi_1^w \geqq 0$. This can only be true if $\psi_2^w/\phi_1^w > 0$, which in turn implies that $\phi_2^w(w) = 1$.[3] □

A numerical example will now be presented to illustrate how equilibrium employment, hours worked, and consumption are potentially related to the lump-sum costs of employment. To construct the example, let $U(c) = \ln(c)$, $V(l) = -l^2$, $f(\phi_t^w l_t, K) = (\lambda_t \phi_t^w l_t)^{0.7} - \kappa_t$ where $\kappa_1 = 0$, $\lambda_1 = 1$, $\kappa_2 = 0.85$, $\lambda_2 = 2$, and $\beta = 0.5$. Given this representation of the economy, table 1 reports the values of the seven endogenous variables for various values of the lump-sum employment costs. It is interesting to note that as the lump-sum costs of hiring increase, the equilibrium employment rate falls, while the number of hours worked per employed agent rises. On the one hand, as lump-sum hiring costs increase, there is clearly a *substitution* effect which will operate to reduce the number of employees hired by increasing the number of hours worked by the remaining workers. On the other hand, since $V(\cdot)$ is a decreasing concave function, there are limits to the firm's ability to persuade a few workers (or one worker) to do all the production in the economy. The latter observation is reflected in the fact that consumption falls as the lump-sum hiring costs are increased, implying that it isn't optimal to make the remaining employed agents pick up enough extra hours to compensate for the cut in the

3 Suppose preferences were written as $U(c_1) + V(l_1) + \beta[U(c_2) + V(l_2)] + \epsilon W(l_1, l_2)$ for $\epsilon \geqq 0$. Let $\phi_2^w(w, \epsilon)$ and $\phi_2^w(u, \epsilon)$ be the resulting optimal choices for period-two employment. If hiring costs are positive, proposition 8 implies $\phi_2^w(w, 0) > \phi_2^w(u, 0)$. By the continuity of optimal solution, employment at the individual level could display persistence (i.e., $\phi_2^w(w, \epsilon) \geqq \phi_2^w(u, \epsilon)$) for some $\epsilon > 0$ even if $W_{12} < 0$. This stands in contrast to the results of proposition 3.

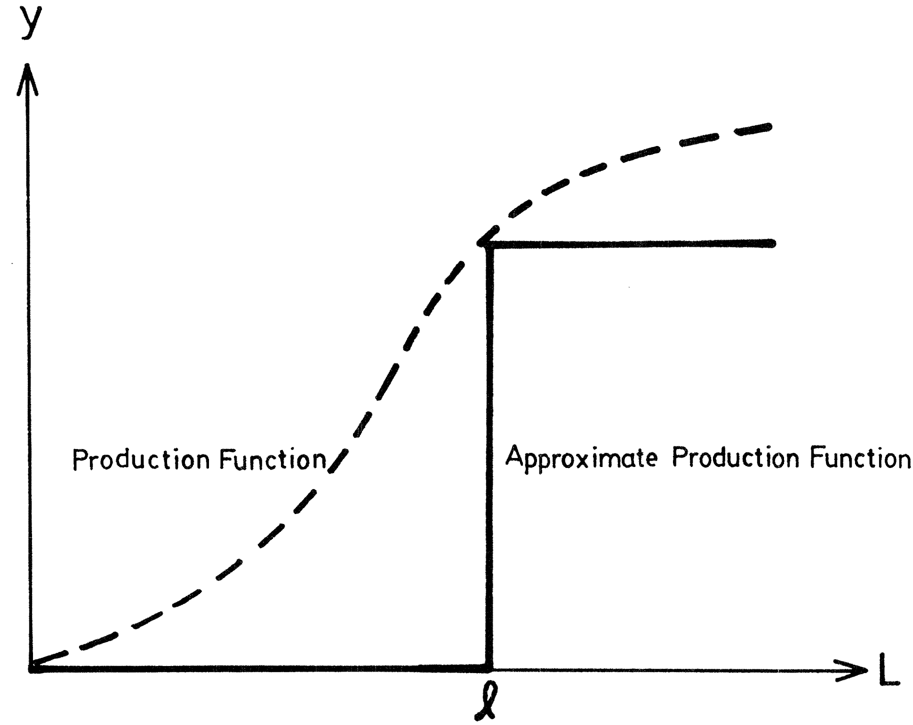

FIGURE 1

labour force. Thus a natural trade-off arises between the equilibrium employment rate and hours worked per employee, owing to the assumed non-convexity in the labour-hiring decision.

Recall that in the fourth and fifth sections of the paper the amount of labour an agent could supply was restricted to be an element in the two-point set $\{0, l\}$. This assumed indivisibility in labour à la Rogerson (1985) may be thought of as representing a crude approximation to a production technology which is not concave in labour input over the domain $[0, l]$, such as represented in figure 1. The non-concave region of the production function could reflect 'starting or warming-up' costs associated with the production activity. This type of phenomenon, however, is perhaps better characterized by the approximate technology illustrated in figure 2; either an agent works at *least l* or he doesn't work at all. Such an extension can be a fruitful ingredient in modelling labour market adjustment along both the extensive and intensive margins. This idea will be incorporated into the current setting by letting agents in period t work either the amount $l_t \geqq l$ or not at all. Thus, the representative agent's lifetime utility function $U(c_1, c_2, l_1, l_2)$, as specified by (38), is now defined over the non-convex set $R_+ \times R_+ \times \Lambda \times \Lambda$, where $\Lambda \equiv \{0\} \cup [l, \infty)$. Another useful ingredient in modelling both margins of labour

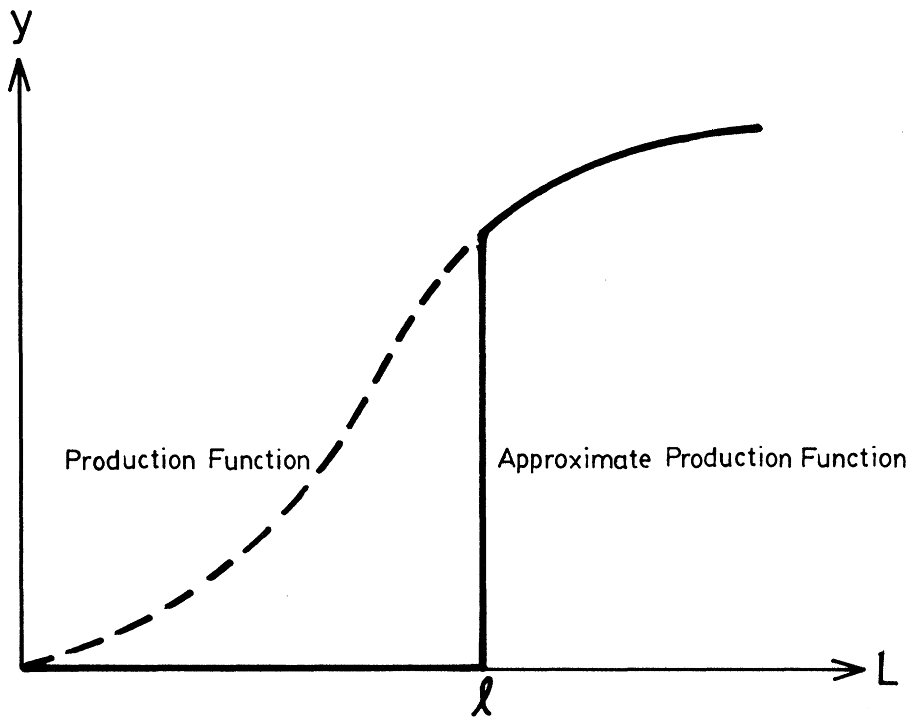

FIGURE 2

force participation is to incorporate some notion of 'on-the-job training.' It may be the case that an individual's productivity is directly affected by work experience. In particular, in the current two-period setting it will be assumed that agents who worked in period one are more productive in the second period by a factor of $\lambda > 1$ – the skill factor – over those who don't have work experience.[4,5] Last, in order to focus more sharply on the effects of the factors currently under consideration, lump-sum hiring costs will be dropped from the analysis; that is, $\gamma = 0$.

Incorporating these ingredients into the assumed economic environment leads to a competitive equilibrium – here a determination of c_1, c_2, $\phi_1{}^w$, $\phi_2{}^w(w)$, $\phi_2{}^w(u)$, l_1, $l_2(w)$, and $l_2(u)$ – which is described by the solution to the system of equations shown below

$$c_t = f(L_t, K) \qquad \text{(cf. (2) and (3))} \tag{44}$$

4 This appears to be the simplest method of employing an intertemporal heterogeneity in the technology. More complicated formulations could be imagined.

5 Grilli and Rogerson (1986) also produce a model where previously employed agents are more productive than are unemployed agents. Hansen and Sargent (1987) display a model of straight-time and overtime shift work where agents may work either zero, straight-time, or straight-time plus overtime.

$$V(0) - V(l_1) + \beta\{E[V(\cdot\, 2) \mid l_1 = 0] - E[V(\cdot\, 2) \mid l_1 \geqq l]\}$$
$$= U'(c_1)f_1(L_1, K)l_1 + \beta U'(c_2)\{f_1(L_2, K)[\phi_2{}^w(w)\lambda l_2(w) - \phi_2{}^w(u)l_2(u)]\} \text{ (cf. 11)} \tag{45}$$

$$\beta[V(0) - V(l_2(w))] + \psi_2{}^w/\phi_1{}^w \geqq \beta U'(c_2)f_1(L_2, K)\lambda l_2(w)$$
$$\text{(cf. (12), with equality if } \phi_2{}^w(u) > 0) \tag{46}$$

$$\beta[V(0) - V(l_2(u))] + \psi_2{}^u/(1 - \phi_1{}^w) \geqq \beta U'(c_2)f_1(L_2, K)l_2(u)$$
$$\text{(cf. (13), with equality if } \phi_2{}^w(u) > 0) \tag{47}$$

$$-V'(l_1) \geqq U'(c_1)f_1(L_1, K) \qquad \text{(cf. (14), with equality if } l_1 > l) \tag{48}$$

$$-V'(l_2(w)) \geqq U'(c_2)f_1(L_2, K)\lambda$$
$$\text{(cf. (15), with equality if } l_2(w) > l) \tag{49}$$

$$-V'(l_2(u)) \geqq U'(c_2)f_1(L_2, K)$$
$$\text{(cf. (16), with equality if } l_2(u) > l) \tag{50}$$

[Note that the complementary slackness conditions (17) and (18) also hold here and recall that $L_1 = \phi_1{}^w l_1$ and $L_2 = \phi_1{}^w\phi_2{}^w(w)\lambda l_2(w) + (1 - \phi_1{}^w)\phi_2{}^w(u)l_2(u)$].

Once again, in general it is difficult to say much about the solution to the above system of equations. It can be discerned, though, that an interesting phenomena is permitted to emerge. That is, it is possible to have competitive equilibria where in the second period some agents, the skilled ones, are working overtime (i.e., $l_2(w) > l$) while some unskilled agents are unemployed. Also, it is fairly easy to establish that experienced workers have seniority in the sense that first, they will never be unemployed so long as less experienced workers are employed and second, they will never be laid off while other experienced workers are doing overtime. This fact is stated more formally in the proposition below.

PROPOSITION 9. *$\phi_2{}^w(w) < 1$ only if $\phi_2{}^w(u) = 0$ and $l_2(w) = l$.*

Proof. The fact that $\phi_2{}^w(w) < 1$ only if $\phi_2{}^w(u) = 0$ is easily shown by utilizing arguments similar to those employed in establishing earlier propositions. That $\phi_2{}^w(w) < 1$ only if $l_2(w) = l$ follows from noting, first, that the efficiency condition for $\phi_2{}^w(w)$ dictates that if $\phi_2{}^w(w) < 1$ then

$$[V(0) - V(l_2(w))] = U'(\cdot\, 2)f_1(\cdot\, 2)\lambda l_2(w). \tag{51}$$

Second, the efficiency condition governing $l_2(w)$ necessitates that

$$-V'(l_2(w)) \geqq U'(\cdot\, 2)f_1(\cdot\, 2)\lambda, \tag{52}$$

with this equation holding with equality if $l_2(w) > l$. Third, since $V(\cdot)$ is a

TABLE 2

λ	ϕ_1^w	l_1	$\phi_2^w(w)$	$\phi_2^w(u)$	$l_2(w)$	$l_2(u)$
7.2	0.61	1.45	1.00	0	1.45	*a*
8.2	0.73	1.45	1.00	0	1.49	*a*
9.2	0.87	1.45	1.00	0	1.52	*a*
10.2	1.00	1.45	1.00	0	1.56	*a*

a Value not relevant here

concave function, it follows that

$$[V(0) - V(l_2(w))] < -V'(l_2(w))l_2(w),$$

so both equations (51) and (52) can hold simultaneously only if the latter one remains slack. □

Finally, to establish that it is possible to have skilled workers doing overtime while unskilled ones are unemployed, table 2 reports the solution values to the above system of equations (44)–(50), (17), and (18) for the six labour market variables over a range of values for the skill factor when the economy is represented as follows: $U(c) = c$, $V(l) = -l^2$, $f(L, K) = (L)^{0.7}$, $\beta = 0.5$, and $l = 1.45$. Note that in this example a seemingly implacable form of structural unemployment is displayed; there are two types of agents in the economy here, those who are permanently employed and those who are permanently unemployed.

CONCLUSIONS

An equilibrium model of unemployment was presented here. The economic environment was postulated to be such that there were non-convexities present in either tastes or technology. It was shown that environments with such non-convexities were capable of displaying interesting labour market phenomena. Given the adopted setting, optimal labour contracting always resulted in a certain fraction of the population being unemployed. It was demonstrated that equilibria could be generated where those currently not working had higher probabilities of being unemployed in the future than those currently employed. In fact, a phenomenon resembling structural unemployment could occur where those currently unemployed remained permanently so. Such non-convexities also could allow for a simultaneous determination of both the extensive and intensive margins of labour force participation. It was possible to have certain agents working overtime while others were unemployed. Additionally, equilibria exist where old workers have seniority in the sense that they are never laid off so long as either new workers are being hired or any agents are working overtime.

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