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EXPECTATIONS, THE EXCHANGE RATE, AND THE CURRENT ACCOUNT

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A critical examination is undertaken of the relationship between the exchange rate and the current account in a small open economy. Theoretically, the correlation between the exchange rate and the current account seems to be ambiguous. In particular, the association between movements in the exchange rate and the current account is likely to depend in an essential manner on the nature of exogenous disturbances affecting the two variables simultaneously. Lastly, the question of the role of an optimal monetary policy and the choice of an exchange rate regime in an uncertain environment is raised.

1. Introduction

What determines the value of a nation's exchange rate and its current account? Popular stories link the strength of a country's currency with the state of its current account. A high foreign exchange rate is thought of as being tied to a current account surplus. However, this relationship does not appear to hold for all countries and time periods.

This paper investigates the relationship between the current account and the exchange rate for a small open economy which bears some uncertainty about its future real income, level of government spending, and monetary policy. In this environment current circumstances and expectations about the future play a key role in the money demand decision that governs the evolution of the exchange rate and the consumption–savings decision that determines the current account. A prediction of the model which is constructed is that the correlation between the exchange rate and the current account depends on the source of exogenous disturbance. For example, an unanticipated improvement in current real income will be associated with an exchange rate appreciation and an improvement in the current account. This would seem to be a confirmation of the popular story. By contrast, if agents are led to believe that future real income will be higher, an exchange rate appreciation will be accompanied by a worsening of the current account.

*The substance and form of this paper have been influenced by discussions with Robert King and Alan Stockman, who have been very helpful as well as generous with their time. Of course, the responsibility for any shortcomings in this paper is solely my own.

Consequently, a conclusion of this paper's model is that empirical studies which do not distinguish between sources of underlying exogenous shocks to the economy will find relationships between the current account and the exchange rate which are not stable over time or across countries. In addition, the neoclassical model presented here predicts substantially different responses of the current account and the exchange rate to various exogenous shocks than some recent studies. These differences arise principally from the incorporation of forward-looking behavior of economic agents in both their consumption–savings and money demand decisions. Lastly, in a rather limited context, the question of an optimal monetary policy and exchange rate regime in un uncertain environment is broached.

2. The basic model

Consider the following model of a small open economy with a life span of two periods[1] that has a flexible exchange rate. All individuals in this economy are identical and consume only one good, c, which must be imported from abroad. The imports are financed by sales of an export good, X, with which the economy is naturally endowed. There are no impediments to international trade and, since the economy under discussion is a small open one, the actions of domestic residents can have no impact on the international terms of trade, ρ, by which is meant the relative price of exports in terms of imports. Domestic residents can also freely participate on an international bond market. The bonds are denominated in terms of the imported good so that a bond being bought in the first period for a unit of c can be redeemed in the second period for $1+r^*$ units of c, with r^* being the fixed international real rate of return. Lastly, the domestic government issues a currency which is held solely by nationals. No foreign currency is held by domestic residents.

Since all individuals in this economy are identical, the model can be analyzed using the construct of the representative individual whose lifetime utility function, $\underline{U}(\cdot)$, has the form

$$\begin{gathered}\underline{U}=U(c_1)+\beta U(c_2),\\ 0<\beta<1,\quad U'(c_t)>0\quad\text{and}\quad U''(c_t)<0,\qquad t=1,2,\end{gathered}\tag{1}$$

where the subscript t attached to a variable indicates that its value in time t is being discussed. Also, it happens that this individual's utility function is

[1] The restriction of the model to two periods, viz. a current and future one, is for technical convenience only. In effect, many future periods are being proxied for by a single future period. This procedure does not seem to impose any severe restrictions on the analysis undertaken. An extension to a model with many future periods would seem to involve only technical, and not economic, considerations.

characterized by decreasing absolute risk aversion.[2] This implies that

$$\partial-[U''(c_t)/U'(c_t)]/\partial c_t<0, \qquad t=1,2. \tag{2}$$

The representative individual is endowed with X_t units of the export good in period t. In this period the individual's real income, y_t, expressed in units of c, is simply $\rho_t X_t$. However, in the first period the individual does not know his future real income, y_2, with certainty. In particular, it is assumed that y_2 is a random variable of the following kind:

$$y_2=\bar{y}_2+\xi_2, \tag{3}$$

where $\bar{y}_2$ is a positive constant and ξ_2 is a continuously distributed independent random variable with a zero mean. In addition, ξ_2 is distributed in such a fashion that y_2 is always positive. Individuals are fully cognizant about the relevant facts concerning future income's probabilistic nature.

Now, individuals are postulated to hold money, M, so as to economize on the transactions costs of exchange. A certain fraction, v, of individual's real income, y, is absorbed in transactions costs. Letting P represent the nominal price of the import good, this fraction, v, is assumed to be a convex function of the ratio of money held by the agent to his nominal income, Py. By increasing his holding of money, the individual can cut down on that fraction of real income which is being absorbed in transactions costs. However, as the ratio of money, M, to nominal income, Py, rises, the reduction in the proportion, v, of real income brought about by holding an extra unit of money is reduced. In other words, for a given level of nominal income, there are decreasing returns to holding money. Since the proportion of a person's income absorbed in transcations costs is always likely to be relatively small, the variable v is assumed to be less than unity in value. In other words, one has[3]

$$\begin{aligned} v_t=v(M_t/P_t y_t) \quad \forall t=1,2 \quad &\text{with} \quad v'<0, \\ v''>0 \quad &\text{and} \quad 0<v<1. \end{aligned} \tag{4}$$

[2]Decreasing absolute risk aversion implies that, as an individual's wealth increases, the odds he will demand to engage in a small bet of a fixed size will decrease. See Sandmo (1968) for a more complete discussion of this assumption and the implications it has on savings and asset selection under uncertainty. Finally, note that decreasing absolute risk aversion implies that $U'''(c_t)>0$ for all t.

[3]This manner of introducing money into the model turns out to be quite convenient to handle. There are, of course, other ways of modeling money in international finance models such as the cash-in-advance constraints employed by Stockman (1980). The message being emphasized in the present paper is that individuals hold money because it yields some real benefits and the transactions costs specification adopted here seems adequate for this purpose. None of the results stressed in this paper appear to be particularly sensitive to the way money has been modeled. Also, the manner in which money has been modeled here is similar in style to that used by Dornbusch and Frenkel (1973).

The government in this small open economy undertakes real expenditures of the amount g_t in period t. These expenditures provide no utility for the public. The only means the government has to finance these expenditures are money creation or taxes. Thus, if M_t^s is defined as the amount of government issued currency in existence at time t and τ_t as the real value of taxes then, the government's budget constraints for the two periods appear as

$$M_1^s = P_1(g_1 - \tau_1), \tag{5}$$

$$M_2^s - M_1^s = P_2(g_2 - \tau_2). \tag{6}$$

In addition, g_2 is assumed to be a non-negative continuously distributed independent random variable. Specifically,

$$g_2 = \bar{g}_2 + v_2,$$

where $\bar{g}_2$ is a positive constant and v_2 is a random variable with mean zero. Individuals know these facts about g_2, including the probability density function governing v_2.

Lastly, individuals know that the second period money supply, M_2^s, is a random variable distributed as follows

$$M_2^s = \mu_2 M_1^s(1+\varepsilon_2), \tag{7}$$

where μ_2 is a known positive constant and ε_2 is a continuously distributed independent random variable of a known form with zero mean. Assume that μ_2 is initially equal to unity and that ε_2 is distributed in such a manner so that M_2^s will always be positive. Here, μ_2 will be taken to be a government policy variable and ε_2 to reflect a random element in the money supply which is beyond the government's control.[4]

3. The individual's optimization problem

The representative agent's first period constrained maximization problem is shown below with the choice variables being c_1 and M_1,

$$\max U(c_1) + \beta \mathrm{E}[U(c_2)], \qquad \text{subject to} \tag{8}$$

$$\begin{aligned} c_2 = (1+r^*)\{(1 - v(M_1/P_1 y_1))y_1 - \tau_1 - c_1 - M_1/P\} \\ + (1 - v(M_2/P_2 y_2))y_2 - \tau_2 - (M_2 - M_1)/P_2. \end{aligned} \tag{9}$$

[4] For the results contained in this paper, it doesn't matter whether ε_2 reflects an uncontrollable stochastic element in the money supply or, instead, if it reflects what individuals perceive as a randomness in the government's decision-making process.

This optimization problem can be simplified to an extent by observing the fact that when the second period occurs the individual will choose to hold money, M_2, in an optimal fashion, implying that

$$v'(M_2/P_2y_2) = -1. \tag{10}$$

As a result, one obtains

$$M_2 = kP_2y_2 \quad \text{where} \quad k \equiv v'^{-1}(-1). \tag{11}$$

By substituting (9) and (11) into (8) and carrying out the maximization routine, the following first order conditions arise:[5]

$$U'(c_1) - \beta(1+r^*)\mathrm{E}[U'(c_2)] = 0, \tag{12}$$

$$\beta\mathrm{E}[U'(c_2)(1/P_2 - v'(M_1/P_1y_1)(1+r^*)/P_1 - (1+r^*)/P_1)] = 0. \tag{13}$$

The economic transliteration of (12) is easy to provide. It says that the individual should save until the loss in utility resulting from a reduction in current consumption due to shifting a small amount of resources into bonds is equal to the discounted expected gain in utility due to the possibility of increased future consumption.

The economic interpretation of (13) is not so obvious. This equation can be written as

$$-v'(M_1/P_1y_1) = \mathrm{E}[\pi + r^*]/(1+r^*) + \operatorname{cov}(U'(c_2), \pi)/(1+r^*)\mathrm{E}[U'(c_2)]$$

with

$$(P_2 - P_1)/P_2 \equiv \pi. \tag{14}$$

The left-hand side of this equation expresses money's marginal product while the right-hand side can be thought of as representing the opportunity cost of holding money. The opportunity cost of holding money is constituted of two components. The first term is the expected cost of holding money, or the discounted expected sum of the rate of depreciation on real balances, π, and the real interest rate, r^*. To see that this is the case, imagine that the

[5]The second order condition for the above optimization problem implies that the subsequent expression for Ω should be positive,

$$\begin{aligned}\Omega = &\{U''(c_1) + \beta(1+r^*)^2\mathrm{E}[U''(c_2)]\}\{\beta\mathrm{E}[U''(c_2)(1/P_2 - v'(M_1/P_1y_1)(1+r^*)/P_1 \\ &- (1+r^*)/P_1)^2] - v''(M_1/P_1y_1)((1+r^*)/P_1^2y_1)\beta\mathrm{E}[U'(c_2)]\} \\ &- \{\beta^2(1+r^*)^2\mathrm{E}[U''(c_2)(1/P_2 - v'(M_1/P_1y_1)(1+r^*)/P_1 - (1+r^*)/P_1)]^2\} > 0.\end{aligned}$$

individual desires to hold an extra unit of real balances in period one. To finance this purchase of money the individual could issue a unit real bond in the first period which pays off $1+r^*$ units of the consumption good in the second period. But $1+r^*$ isn't the real cost the individual incurs in the second period in order to increase his first period real money holdings by a unit. This is because the individual now has P_1 units of nominal balances in the second period which he has carried over from the first period, and that can be sold off for P_1/P_2 units of the consumption good. Thus, the second period real cost the individual incurs is $1+r^*-P_1/P_2$. Now, take the expected value of this term and discount it. One gets that the expected real cost of holding an extra unit of cash balances in the first period (expressed in first period terms) is $\mathrm{E}[\pi+r^*]/(1+r^*)$.

The second component in (14) is a risk premium term. This term would be equal to zero in either a deterministic world or one where individuals were risk neutral. In these two situations the term $\mathrm{E}[\pi+r^*]/(1+r^*)$ represents the opportunity cost of holding money. It can be shown (see appendix A) that the risk premium term is positive since the marginal utility of second period consumption and the second period's price level, and hence $\pi+r^*$, covary directly with each other. Thus, in an uncertain world the risk averse individual will hold money in the first period such that its return, $-v'(M_1/P_1y_1)$, is greater than the expected cost of holding money, $\mathrm{E}[\pi+r^*]/(1+r^*)$. This implies that the individual will hold less money in the first period under uncertainty than he would in a certain setting. The intuitive reason for this is that in the model's general equilibrium money is a risky asset to hold. It is risky to hold because times when second period real income, y_2, and hence consumption, c_2, are low also correspond to times when the second period's price level, P_2, and thus the cost of carrying money over from the first into the second period, are likely to be high. Because of this risk inherent in holding cash balances, individuals demand that money yields a return $-v'(M_2/P_1y_1)$ greater than its expected cost $\mathrm{E}[\pi+r^*]/(1+r^*)$.

From the above discussion, it is clear that the opportunity cost of holding money is the amount of real resources that a person would be willing to pay now in order to rent a unit of real cash balances for the duration of the first period. Such a rental agreement sheds the risk an individual normally incurs when holding a unit of real cash balances in the first period. This point has been made by Fama and Farber (1979).

Lastly, eqs. (9), (12) and (13) define implicitly the individual's first period consumption and demand for money functions. Many simple monetary approach models of the exchange rate, such as Bilson (1978), tend to express the demand for real cash balances as a neat function of $\mathrm{E}[\pi+r^*]$ and y_1. It should be noted that here such a simple functional relationship cannot be obtained unless the model is deterministic or the individual is risk neutral. Then, the inversion of (14) would give

$$M_1/P_1 = v'^{-1}(-(\pi + r^*)/(1 + r^*))y_1. \tag{15}$$

In general, the type of demand for money functions used by the simple monetary approach models does not impose any serious limitations on the analysis undertaken. However, in the last section of this paper the role of monetary policy as a mechanism to allow money to be used efficiently will be discussed. Here, it will be important to recognize explicitly how risk affects individuals' utilization of cash balances. As has already been alluded to, the negative covariation between the price level and real income leads to an underutilization of cash balances. A question to ask is: can monetary policy remedy this situation?

4. The model's general equilibrium in the first period

The model's first period general equilibrium will now be discussed. To begin with, if the money market is to clear always, then money demand must equal money supply each period. Formally,

$$M_t = M_t^s \quad \forall t = 1, 2. \tag{16}$$

By substituting the above condition for money market equilibrium and expression (11) into the individual's first order conditions (12) and (13), one obtains eqs. (17) and (18),

$$U'(c_1) - \beta(1 + r^*)\mathrm{E}[U'(c_2)] = 0, \tag{17}$$

$$\mathrm{E}[U'(c_2)(P_1 k y_2/(1 + r^*)M_2^s - v'(M_1^s/P_1 y_1) - 1)] = 0. \tag{18}$$

Now, by using the government's budget constraints (5) and (6) in eq. (9) while also making use of (11) and (16), it can be seen that

$$c_2 = (1 + r^*)\{(1 - v(M_1^s/P_1 y_1))y_1 - c_1 - g_1\} + (1 - v(k))y_2 - g_2. \tag{19}$$

There are no direct wealth effects from holding money in the model's general equilibrium.[6] Money does have an indirect wealth effect, however, since it

[6]The reason why there is an absence of a direct wealth effect from holding money on consumption can be expanded upon further. In the model's general equilibrium, the real value of cash balances is exactly offset by the real expense individuals incur from holding them. This expense includes the capital loss individuals suffer from holding money at the end of the second period when, so to speak, the real value of their cash balances is completely written off. The model allows for no mechanism by which individuals can rid themselves of the perceived burden from holding real cash balances at the end of the second period when the world ends. For instance, in a more general model such a mechanism would exist if there was a third party who lived on in a third period and who would be willing to pay individuals a certain amount in the second period for the ownership rights to their nominal cash balances in the third period. The

allows individuals to economize on their transactions costs of exchange and thereby improves their real disposable income. This is apparent from the efficiency conditions (17) and (18) and the economy-wide budget constraint (19) in which money enters only via the transactions cost terms, or through the v's.

To see this more clearly, imagine that there is no uncertainty in the model. Then (17), (18) and (19) imply implicit functions for c_1 and P_1 of the following form:

$$c_1 = c_1(w, r^*), \qquad P_1 = P_1(y_1, y_2, M_1^s, M_2^s, r^*), \tag{20}$$

$$\text{with} \quad w \equiv (1 - v(M_1^s/P_1 y_1))y_1 - g_1 + \{(1 - v(k))y_2 - g_2\}/(1 + r^*).$$

Here, money affects consumption only insofar as it affects disposable wealth, w, through its impact on transactions costs, or again through the v's.

Two more equations are needed to specify completely general equilibrium in the first period. First, if the law of one price always holds it must be the case that

$$P_1 = e_1 P_1^*, \tag{21}$$

where e_1 is the first period's exchange rate, or the domestic currency price for a unit of foreign currency and where P_1^* is the foreign nominal price for a unit of the consumption good. Secondly, the current account balance, CB_1, for the first period expressed in terms of c is

$$CB_1 = (1 - v(M_1^s/P_1 y_1))y_1 - c_1 - g_1. \tag{22}$$

It can be seen that by substituting eq. (19) into (17) and (18), two implicit functions arise that define a solution for c_1 and P_1. By using these (implicit) solutions for c_1 and P_1 in (21) and (22), respectively, the value of today's exchange rate, e_1, and the current balance, CB_1, can be determined. Thus, eqs. (17), (18), (19), (21) and (22) implicitly define a solution for c_1, P_1, e_1 and CB_1.

5. Changes in expectations about the future

The likely impact effects of changes in expectations about the future money supply, real income, and government expenditure on today's exchange

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ability to sell one's cash balances off at the end of the second period would result in a direct wealth effect from holding money. However, as a person's lifespan is lengthened, this kind of wealth effect from holding money becomes less and less important, and in the limit for an infinitely-lived individual vanishes.

rate and the current account balance are fairly easy to analyze. To undertake this exercise, consider changes in $E[M_2^s]$, $E[y_2]$, and $E[g_2]$ due to shifts in μ_2, $\bar{y}_2$, and $\bar{g}_2$. The impact of shifts in these variables on the current price level, P_1, and the current level of consumption, c_1, can be determined through the use of eqs. (17), (18), and (19). Since technically this is a standard, albeit somewhat onerous, exercise in comparative statics, providing little economic insight, the details of this procedure have been relegated to appendix B. The results are

$$\begin{aligned} &\partial c_1/\partial \bar{y}_2 > 0, \qquad \partial P_1/\partial \bar{y}_2 < 0, \\ &\partial c_1/\partial \mu_2 < 0, \qquad \partial P_1/\partial \mu_2 > 0, \\ &\partial c_1/\partial \bar{g}_2 < 0, \qquad \partial P_1/\partial \bar{g}_2 > 0. \end{aligned} \tag{23}$$

Consider first the impact effect of an expected increase in the future money supply. As can be seen from (23), this leads to a rise in the current price level since $\partial P_1/\partial \mu_2$ is positive. The reasoning for this result is direct. An increase in the expected future money supply leads people to believe that the future price level, and thus the cost of holding money, will be higher. Consequently, people desire to hold less real balances currently which, given the fixed nominal stock of money, necessitates an increase in the present price level. As a result, the increase in the expected future money stock causes an immediate depreciation of today's exchange rate, e_1. This follows from the law of one price (21) which, when differentiated with respect to μ_2, yields

$$\partial e_1/\partial \mu_2 = (1/P_1^*)\partial P_1/\partial \mu_2 > 0.$$

This expression is positive because, as just discussed, $\partial P_1/\partial \mu_2$ is positive. The fact that an increase in the expected future money supply should lead to a depreciation of the current exchange rate, for the reason stated above, is a ubiquitous conclusion of simple monetary approach models, typified by Bilson (1978).

It happens that current consumption falls in response to a rise in the expected future money supply. The intuitive reason for this result is easy to provide. As has been mentioned, the increase in $E[M_2^s]$ leads people to believe the cost of holding money will rise. Thus, they try to economize today on their holdings of real cash balances. This means, though, that they must incur greater transactions costs which reduce their current real disposable income, $(1-v(\cdot))y_1$, and consequently their expected real wealth. Thus, current consumption drops. The effect on today's current account balance, CB_1, can be discovered through formulae (22) and (23). It happens that

$$\partial CB_1/\partial\mu_2 = v'(1)(M_1^s/P_1^2)\partial P_1/\partial\mu_2 - \partial c_1/\partial\mu_2 < 0, \quad \text{where}$$

$$v(1) \equiv v(M_1^s/P_1 y_1).$$

A movement toward a current account deficit now arises because the impact of a reduction in current real disposable income is spread out over expected consumption in both periods so that current consumption does not fall by as much as current real disposable income. This effect on the current account balance of an expected future monetary expansion is likely to be small, since one would expect the increase in today's transactions costs, and consequently the fall in real disposable income, due to an increase in the expected rate of inflation to be of relatively minor importance.

This result contrasts with Dornbusch and Fischer (1980) who argue that an increase in the future nominal quantity of money should lead to a current account surplus, at least until until the time the money stock expansion actually occurs, whereupon a current account deficit ensues. Their result follows from the fact that they include real balances in their definition of real wealth upon which households base their consumption–savings decisions. The anticipated monetary expansion in their model leads to a rise in the expected rate of inflation which causes households to reduce their real balances. The drop in real balances makes people less wealthy, causing households to save more and thus leads to a current account surplus. This channel of effect on the current account is absent in the current model since, in general equilibrium, there are no direct wealth effects from holding money, per se. By adopting a choice-theoretic approach in the present paper, it is hoped that potential misspecifications of agents' consumption–savings decision rules due to ad hoc definitions of wealth are avoided.

Eq. (23) illustrates that an upward shift in the expected value of y_2 leads to a drop in the current price level, P_1. Using the law of one price, it then happens that

$$\partial e_1/\partial\bar{y}_2 = (1/P_1^*)\partial P_1/\partial\bar{y}_2 < 0.$$

An increase in expected future income leads to an appreciation of today's exchange rate because agents believe that the future price level will be lower, ceteris paribus. This has a deflationary effect which makes the perceived opportunity cost of holding real balances today lower. Resultantly, the current demand for real balances rises which, given the fixed nominal stock of money, leads to a fall in the present price level and an appreciation of the exchange rate. This effect is standard and is present in many monetary approach models, such as Bilson (1978).

Lastly, as is evident from (23), current consumption rises consequent to an increase in expected future income. Rather than consume the expected increase in future income solely in the second period, individuals instead desire to smooth out expected consumption over the two periods, and this leads to an increase in current consumption. This consequence of the model is congruent with any theory that postulates that present consumption should be based on permanent income, or some related wealth concept.

From (22), it can now be seen that an increase in expected future income will be likely to have a negative impact on the current account since

$$\partial CB_1/\partial \bar{y}_2 = (v'(1)M_1^s/P_1^2)\partial P_1/\partial \bar{y}_2 - \partial c_1/\partial \bar{y}_2 < 0.$$

The above discussion explains that the first term is positive. This reflects the fact that as the current price level, P_1, falls, real balances in this period rise, and consequently transactions costs decrease, so that real disposable income, $(1-v(1))y_1$, increases. An improvement in today's real disposable income tends to improve the current account balance. However, this effect is likely to be small since one can safely assume that the reduction in transactions costs due to a fall in the price level is likely to be relatively minor. The second term in the above equation is negative and is likely to be dominant for the above reason. Therefore, an increase in the second period's expected real income is likely to lead to a proclivity toward a current account deficit as people borrow more on the international bond market in order to finance an increase in current consumption.

Lastly, consider the impact effect of an increase in expected future government spending on today's exchange rate and the current account. Assume that this increase in future government spending is to be financed by an increase in future taxation.[7] To begin with, current consumption, c_1, falls with the increase in expected future government spending as is manifested by (23). Intuitively, this is because individuals are cognizant of the fact that an increase in expected future government spending must be financed by a rise in future taxes. Instead of reducing consumption solely in the future period when the tax hike occurs, people prefer to spread out the reduction in consumption across both periods. Thus, current consumption falls. Current

[7]As can probably be discerned, the period in which the government collects the taxes to finance the expected future increase in government spending is largely irrelevant for today's consumption. The individual would be equally pleased if the government collected in taxes today the discounted value of the expected increase in government expenditure and then invested these tax proceeds in foreign real bonds which would be used to finance the increased future government spending when it occurs. Loosely speaking, all the individual cares about is the expected increase in the present value of government spending, or equivalently the expected increase in the present value of taxes. Thus in this model Ricardian equivalence holds. A more detailed discussion of Ricardian equivalence in international finance models is contained in Stockman (1983). As is evident from (22) in this model, the period in which government actually collects the taxes to finance its increased expenditure is irrelevant for today's current account.

consumption is not based on current income but on expected disposable wealth, and when calculating disposable wealth individuals net out the expected present value of government spending. For the deterministic version of the model this is shown clearly by (20).

Today's price level, P_1, increases with the upward shift in expected future government spending. From (11) and (16) it can be seen that, as modeled, the increase in expected future government spending has no impact on the expected future price level, P_2, since it doesn't affect either future income, y_2, or the money stock, M_2^s. Thus, changes in the expected cost of holding money cannot be the operational force influencing today's price level. Now, it seems that movements in the risk premium on holding money, which is a component of money's opportunity cost, are the factors influencing today's price level. As disposable wealth falls people become more and more risk averse, and hence less willing to hold money in the first period.[8] That is, the risk premium on holding money, and consequently the opportunity cost of holding money, increases leading to a reduction in the demand for money. Thus, the current price level rises.[9] However, one would expect such movements in the current price level, due to changes in money's risk premium because of increased expected future government spending, to be of quantitatively minor importance. Therefore, this example illustrating how a rise in g_2 influences P_1 via changes in the risk premium is of more theoretical than practical importance.

The impact of a rise in expected future government expenditure on the current account is given by

$$\partial CB_1/\partial \bar{g}_2 = v'(1)(M_1^s/P_1^2)\partial P_1/\partial \bar{g}_2 - \partial c_1/\partial \bar{g}_2 > 0.$$

The first term, which is negative, shows the detrimental effect that a rise in the current price level has on the current account due to increased transactions costs and hence lower disposable income. This term is likely to be small in magnitude, since changes in transactions costs caused by movements in the price level presumably are of secondary importance, and also because the change in the price level itself, $\partial P_1/\partial \bar{g}_2$, is likely to be small.

[8]This follows from the assumption of decreasing absolute risk aversion made earlier in the paper. Sandmo (1968) discusses how this assumption and people's asset selection and saving decisions are interrelated. Appendices B and C discuss the consequences of this assumption for this paper.

[9]Note that alternative ways of modeling money could obviously lead to different scenarios concerning the price level. For instance, suppose that the government has a demand for cash balances that is some function of the level of its current expenditures. Then an increase in future government spending would lead to an increase in the government's demand for real balances in that period, and assuming a constant private sector demand, would cause an increase in the economy's aggregate demand for cash. Consequently, the second period's price level might fall, resulting in a higher return on money, and, therefore, a tendency towards a lower price level in the first period on this account.

The second term, which is dominant, illustrates the beneficial impact a reduction in consumption has on the current account. Thus, an expected increase in future government spending causes a propensity toward a current account surplus today. This is the open economy analogue of Barro's (1981) result that a rise in expected future government expenditure causes a drop in the real interest rate in a closed economy.

6. Some other comparative static results

The implications for today's exchange rate and the current account from changes in the current supply of money and real income are relatively straightforward. To avoid sounding monotonous, the intuition underlying these comparative static results will only be discussed briefly. To begin with, consider an unanticipated permanent increase in the present money supply (i.e., $dM_1^s = dE[M_2^s]$). Such a monetary expansion leads to a once-and-for-all contemporaneous depreciation in today's exchange rate via the law of one price. One gets the standard conclusion that

$$\partial e_1/\partial M_1^s = e_1/M_1^s > 0.$$

Unlike the case of an expected future monetary expansion, the current account balance is unaffected by an unanticipated permanent increase in today's money stock. This is because such a shift in the money stock causes only a once-and-for-all change in the domestic price level. Thus, the expected cost of holding money remains unchanged so that the real quantity of money, and consequently transactions costs and real disposable income, are left unaltered.

A temporary improvement in the first period's real income, y_1, should cause the exchange rate to appreciate. This is because $\partial P_1/\partial y_1$ is likely to be negative. As the first period's income rises, so do transactions costs in this period. In order to mitigate the burden of rising transactions costs, individuals desire to increase their holdings of real money balances so as to cut down on the costs of exchange. Given the fixed money supply, the price level must fall and concomitantly, via the law of one price as expressed by (21), the exchange rate appreciates. That a rise in current income should cause an appreciation of the exchange rate was emphasized by even the earliest monetary approach models, for instance Johnson (1973).

A temporary improvement in current income is likely to have a positive effect on the current account. This is exactly what should be expected. Rather than consume the increase in current income all today, individuals prefer to smooth out consumption over the two periods and thus increase expected consumption in both periods. This action leads to a movement toward a surplus today. Formally,

$$\partial CB_1/\partial y_1 = (1 - v(1)) - \partial c_1/\partial y_1 + \{v'(1)(M_1^s/P_1 y_1) + v'(1)(M_1^s/P_1^2)(\partial P_1/\partial y_1)\} > 0.$$

The last two terms in braces reflect, respectively, the facts that transactions costs increase with a rise in income but decrease with a fall in the price level. Since the net effect of these changes in transactions costs on real disposable income is likely to be small, these last two terms can be safely ignored here. The first term in the above expression illustrates the positive effect that an upward movement in current real disposable income will have on the current account. The second term shows the negative impact an increase in consumption causes. Since agents don't consume all the temporary increase in current income in the first period, the first term will be larger than the second, and a propensity toward a current account surplus will ensue.

Finally, suppose that the improvement in current income was expected to be permanent[10] (i.e., $dy_1 = E[y_2]$). One would expect that if income rose by the same amount in both periods, then a once-and-for-all proportional drop in the price level would occur. An immediate appreciation of the exchange rate results. Formally,[11]

$$\partial e_1/\partial y_1 \simeq -e_1/y_1 < 0.$$

A permanent change in disposable income can be expected to lead to a one-to-one change in consumption each period.[12] Thus, such an income change should not have an impact on the current account balance,[13]

$$\partial CB_1/\partial y_1 \simeq 0.$$

Dornbusch and Fischer (1980), and Rodriguez (1980) both discuss the association between the current account and the exchange rate. It has been

[10]As a benchmark from which to undertake this exercise, assume that $1/\beta = (1+r^*)$. Also assume that $v(1) = v(k)$, albeit because of the terminal nature of the second period this will probably never be the case. If $v(\cdot)$ is always a small number, as expected, then whether or not this last equality holds isn't too important.

[11]This and the next equation hold exactly when the model is deterministic. For small income risks, it holds in an approximate sense as shown. See appendixes B and C for further details.

[12]As $1/\beta$ becomes larger (smaller) than $(1+r^*)$, a permanent change in income has a tendency to increase (decrease) current consumption more than the future consumption, thus creating a propensity towards a current account deficit (surplus).

[13]Note that in this case, where $\partial P_1/\partial y_1 \simeq -P_1/y_1$, the fall in current transactions costs due to the drop in the price level will approximately offset the rise in transactions costs due to increased income. Thus, a fortiori, the effects of changes in transactions costs on the current account can be ignored.

shown in this paper that an increase in current income or a decline in the expected future money stock leads to an improvement in the current account associated with an appreciating exchange rate. However, a rise in expected future income causes the opposite result with a current account worsening being accompanied by an exchange rate appreciation. A once-and-for-all upward movement in the current money supply causes the exchange rate to depreciate, but has no consequences for the current account. In conclusion, it would seem that, theoretically, in this model no general correlation between the exchange rate and the current account exists and that the relationship between these two variables depends upon the relative magnitudes of the underlying exogenous variables, here current and future incomes, levels of government spending, and money supplies, that govern the economy. This conclusion is readily apparent from table 1 which shows the impact effects on the exchange rate and the current account of certain shifts in some of the model's exogenous variables.

Table 1

The effects some exogenous disturbances have on the current account and the exchange rate.

Shock	Current account	Exchange rate
Unanticipated temporary increase in current income	Improvement	Appreciation
Expected increase in future income	Worsening	Appreciation
Unanticipated increase in current income which is expected to be permanent	Neutral	Appreciation
Unanticipated temporary increase in current money stock	Neutral (or slight improvement)	Depreciation
Expected increase in future money stock	Neutral (or slight worsening)	Depreciation
Unanticipated increase in current money stock which is expected to be permanent	Neutral	Depreciation
Unanticipated temporary increase in current government expenditure	Worsening	Neutral (or slight depreciation)
Expected increase in future government expenditure	Improvement	Neutral (or slight depreciation)
Unanticipated increase in current government expenditure which is expected to be permanent	Neutral	Neutral (or slight depreciation)

Dornbusch and Fischer also find the relationship between the exchange rate and the current account to be theoretically ambiguous. But they seem to imply that the popular view is the normal case, which may have to be qualified in certain circumstances to allow for other patterns of association between the two variables. Such a circumstance would be when individuals expect an increase in the future money stock which, as previously discussed, leads to an improvement in the current account and a depreciation of the exchange rate in their model. It may be true that as an empirical proposition the popular view is generally borne out, albeit there is no theoretical presumption for this to be the case. An appealing property of small scale dynamic general equilibrium models, such as the one presented here, is that they are capable of yielding a set of predictions that seem to be tractable to empirical testing.

Basically, the current account is the difference between income and spending in an economy. The present model suggests that spending — here consumption and government spending, the latter of which is assumed to be exogenous — should be empirically modeled in accordance with the rational expectations–permanent income hypothesis, something which Hall (1978), Sargent (1978), Flavin (1981), and others have empirically tested. As is well known in macroeconomics, the rational expectations–permanent income theory places strongly testable restrictions on the forms of equations that can be used to model empirically agents' consumption–savings decision rules. A final caveat should be interjected here. A complete model of the current account should also incorporate, as part of spending, the amount of investment expenditure that an economy undertakes. Empirically modeling investment spending correctly, however, is not an easy matter.

7. A role for a state-contingent monetary policy

The above neoclassical model is also flexible enough to address the issue of the role for an optimal state-contingent monetary policy in a small open economy. To examine this question, imagine the situation where the economy is controlled by a benevolent and omniscient central planner who can decree individual behavior and who can produce currency costlessly. To determine the socially optimum values of c_1 and M_1^s/P_1 the central planner should solve the maximization problem (24) posed below, achieving the first order conditions (25) and (26),

$$\max U(c_1)+\beta \mathrm{E}[U(c_2)], \quad \text{subject to} \tag{24}$$

$$c_2=(1+r^*)\{(1-v(M_1^s/P_1y_1))y_1-c_1-g_1\}+(1-v(k))y_2-g_2,$$

$$U'(c_1)-\beta(1+r^*)\mathrm{E}[U'(c_2)]=0, \tag{25}$$

$$v'(M_1^s/P_1 y_1)=0. \tag{26}$$

In the perfectly competitive economy, the government can adopt a state-contingent monetary policy which would duplicate the results obtained by the centrally planned economy. To see how this can be done, note that eq. (18) can be written as

$$v'(1)=\mathrm{E}[U'(c_2)P_1 k y_2/M_2^s(1+r^*)]/\mathrm{E}[U'(c_2)]-1. \tag{27}$$

Therefore, if $v'(1)$ is to be made equal to zero, the right-hand side of (27) must be made equal to zero. Recall that M_2^s is equal to $\mu_2 M_1^s(1+\varepsilon_2)$ so that in the second period the government can influence the value of M_2^s by manipulating μ_2. Let's now suppose that the government precommits itself to following the monetary policy prescribed below,

$$\mu_2=P_1 k y_2 \mathrm{E}[1/(1+\varepsilon_2)]/(1+r^*)M_1^s. \tag{28}$$

Substituting this formula for μ_2 into (7), and then (7) into (27) yields

$$v'(1)=\mathrm{E}[U'(c_2)/(1+\varepsilon_2)]/\mathrm{E}[U'(c_2)]\mathrm{E}[1/(1+\varepsilon_2)]-1=0.$$

The state-contingent monetary policy outlined above makes the opportunity cost of holding money equal to zero. As was shown earlier, the opportunity cost of holding money is made up of two components, viz. the expected cost of holding money and a risk premium term. Using (7), (11), (16) and (28) an expression for the second period price level associated with the monetary policy can be obtained.

$$P_2=P_1(1+\varepsilon_2)\mathrm{E}[1/(1+\varepsilon_2)]/(1+r^*) \tag{29}$$

As a consequence, the expected cost of holding money, $\mathrm{E}[\pi+r^*]/(1+r^*)$, is zero since

$$\mathrm{E}[\pi]=\mathrm{E}[(P_2-P_1)/P_2]=-r^*.$$

In other words, the expected rate of depreciation on real balances is now equal to minus the rate of interest. This is, of course, Friedman's rule for determining the optimum quantity of money.

Secondly, the risk premium component of the opportunity cost of holding money has been eliminated due to the government's precommitment to the above monetary policy. Recall that the risk premium term in competitive equilibrium arises because the marginal utility of consumption is positively correlated with the cost of holding money. As was previously mentioned, those times when second period real income, and hence consumption are low, and thus the marginal utility of consumption is high, are also likely to correspond to times when the second period price level, and consequently the cost of carrying money over from the first into the second period are high. This makes money a risky asset to hold. But the adoption of the proposed state-contingent monetary policy eradicates the positive covariation between the marginal utility of consumption and cost of holding money. This is most easily grasped by formula (29) for P_2 which doesn't involve second period income, y_2, in it at all. This is not to say that the second period's price level doesn't contain any randomness in it, because it does, as is manifested by the presence of $(1+\varepsilon_2)$ in the numerator of (29). What is important, however, is that this randomness in the price is uncorrelated with income, as it is here.[14]

Precommitment to the above monetary rule sheds the risk from holding money individuals suffer due to unanticipated changes in the second period price level ensuing from random changes in real income and allows individuals to hold money more efficiently. By following the above monetary policy, the undesirable negative correlation between the price level and income is eliminated. Lastly, the above discussion has some implications for the optimal choice of exchange rate regime. In order to carry out the above type of monetary policy which stabilizes the future price level, the government must adopt a flexible exchange rate system. Thus, on this ground alone, a flexible exchange rate system would seem to be preferable to a fixed exchange rate system which doesn't allow the presence of such a monetary policy.[15]

8. Conclusions and extensions

In general, there would seem to exist no unique relationship between the current account and the exchange rate. Within the context of this paper's model, a temporary positive shock to current income causes an exchange rate appreciation plus a current account improvement. However, an expected

[14]The idea that randomness in the price level per se isn't necessarily an undesirable thing was brought to my attention by Robert King. He may, however, frown on the way that his idea has been operationalized in this paper. Finally, the method employed in appendix A can be used to show formally that the proposed state contingent monetary policy results in $\mathrm{cov}(U'(c_2), \pi) = 0$.

[15]Helpman (1981), in an interesting paper, constructs a neoclassical model with cash-in-advance constraints to address the question of the choice of exchange rate regime for an economy. His conclusion is that the choice of exchange rate regimes has no welfare implications for the economy — and hence, presumably, is an irrelevant issue. This result obtains because money has no real effects in his model. Aschauer and Greenwood (1983) extend Helpman's model to allow domestic output to be an endogenous function of domestic labor input and domestic welfare to be dependent upon domestic labor services as well as consumption. They find that the choice of exchange rate regime does have welfare implications for an economy. In particular, a flexible exchange rate regime should be preferred to a fixed one because it allows a nation to pick optimally its rate of inflation. In their deterministic model when the domestic inflation rate is picked optimally $\pi = -r^*$, which is similar to the result obtained in the text above.

improvement in future income leads to an exchange rate appreciation associated with a worsening of the current account. A once-and-for-all increase in today's money stock has no implications for the current account, but will cause a depreciation in the value of the nation's currency. By comparison, an expected rise in the future money supply results in a tendency toward a deficit today accompanied by a depreciation in the exchange rate.

While the focus of this paper has been on the relationship between the current account and the exchange rate, it seems that the general line of argument can be used to obtain some predictions between the current account and the relative price of non-traded to traded goods. For instance, consider extending the model presented by giving the economy a fixed endowment of non-traded goods each period. Then it would seem reasonable to speculate that an expected increase in future real income, say due to an increase in the endowment of the export good, would lead to an increased demand today for both import and non-traded goods. Since the market for non-traded goods must clear domestically, one would expect that today's relative price for non-traded goods would have to rise to keep supply equal to demand. Thus, an increase in expected future income for the above reason would tend to be associated with a propensity toward a current account deficit, a rise in the relative price of non-traded goods, and an exchange rate appreciation.

Contrarily, consider a rise in current real income caused by a temporary improvement in today's endowment of the export good. Again, one would expect the consumption demand for both the import and non-traded goods to increase. Consequently, the relative price of the non-traded good would have to rise so that the market can clear domestically. If individuals are smoothing out consumption over time, the increase in the demand for imports will fall short of the increase in exports so that a proclivity toward a current account surplus will arise. The final result should be an improvement in the current account, linked to a rise in the relative price of non-traded goods, and an exchange rate appreciation. The conclusion to be drawn is that the correlation between changes in the relative price of nontraded goods and movements in the current account (and for that matter the former and the exchange rate), also depends upon the nature of the underlying exogenous shocks simultaneously affecting them. A more complete analysis along these lines is present in Greenwood (1983).

Some recent work in international finance has been concerned with the real causes of exchange rate movements and how these have been associated with a breakdown of purchasing power parity (p.p.p.). For instance, Stockman (1980) shows how changes in the international relative prices of traded goods, the exchange rate, and aggregate price indices are likely to be correlated. Jones and Purvis (1981) discuss how movements in the international relative price of intermediate traded goods can impact on a nation's aggregate price index and exchange rate. The present setup can be extended, as in Greenwood (1983), to allow for a discussion of the relationship between real shocks and the breakdown of p.p.p.

Consider again the above example which discussed how an expected increase in future real income affects today's relative price of non-traded goods. Such a shock to expected future real income could lead to a breakdown in p.p.p. To see this think of the domestic aggregate price index, P, as being some homogeneous function of degree one in the domestic nominal prices of the non-traded good, P_n, and the imported good, P_i. Thus, $P=F(P_n, P_i)$. Due to the homogeneity of $F(\cdot)$ and the law of one price, this price index, P, can be expressed alternatively as $P=eP_i^*F(\rho_n, 1)$, where P_i^* is the foreign price of the imported good and ρ_n is the relative price of non-traded goods. Now, as is commonly done to get a measure of p.p.p., divide the domestic price index by e so as to express it in foreign currency units and then divide the resulting expression by the foreign aggregate price index, P^*. Thus, the measure being used to reflect p.p.p. is $P/eP^*=P_i^*F(\rho_n, 1)/P^*$. Since by assumption the domestic country is a small open economy, all foreign prices, and consequently the foreign aggregate price index, P^*, will be unaffected by any shocks emanating within the domestic economy. Consequently, the expectation of a rise in future domestic income, due to a rise in the endowment of the export good, will lead to a breakdown of p.p.p. since, as can be seen, P/eP^* will rise concomitantly with the rise in the relative price of non-traded goods.

This paper provides an example of a class of models that should have broad applications to problems in international finance: small scale dynamic general equilibrium models that simultaneously determine the exchange rate, relative prices, and real flows such as the current account. Within these models, changes in current and expected future real opportunities and monetary policies have implications for the exchange rate and the current account that typically depend, in an essential manner, on the source of the shock. It seems that research developing empirical strategies aimed at exploiting the set of predictions provided by this class of models could prove to be fruitful.

Appendix A

It can be shown that $\operatorname{cov}(U'(c_2), \pi) > 0$. To begin with,

$$\operatorname{cov}(U'(c_2), \pi) = -P_1 \operatorname{cov}(U'(c_2), 1/P_2) \quad (\text{since } \pi \equiv 1 - P_1/P_2).$$

However, in general equilibrium it is known that

$$P_2 = M_2^s/ky_2.$$

Consequently,

$$\operatorname{cov}(U'(c_2),\pi) = -P_1 k \operatorname{cov}(U'(c_2), y_2/M_2^s)$$

$$= -P_1 k \int_a^b \int_d^e \int_f^h U'(c_2)(y_2/M_2^s - \bar{y}_2\overline{(1/M_2^s)})\phi(y_2)$$

$$\times \gamma(g_2)\psi(M_2^s)\mathrm{d}y_2\mathrm{d}g_2\mathrm{d}M_2^s,$$

where $\phi(\cdot)$, $\gamma(\cdot)$, and $\psi(\cdot)$ are the density functions for y_2, g_2, and M_2^s respectively and where $\bar{x} \equiv \mathrm{E}[x]$. Integrating with respect to M_2^s, one gets

$$\operatorname{cov}(U'(c_2),\pi) = -P_1 k\overline{(1/M_2^s)} \int_d^e \int_f^h U'(c_2)(y_2 - \bar{y}_2)\phi(y_2)\gamma(g_2)\mathrm{d}y_2\mathrm{d}g_2$$

$$= -P_1 k\overline{(1/M_2^s)} \int_d^e \left\{ \int_f^{\bar{y}_2} U'(c_2)(y_2 - \bar{y}_2)\phi(y_2)\mathrm{d}y_2 \right.$$

$$\left. + \int_{\bar{y}_2}^h U'(c_2)(y_2 - \bar{y}_2)\phi(y_2)\mathrm{d}y_2 \right\} \gamma(g_2)\mathrm{d}g_2.$$

Let c_2^* be the value of c_2 that corresponds with the value of y_2^* of y_2 such that

$$c_2^* = (1+r^*)[(1-v(1))y_1 - g_1 - c_1] + (1-v(k))y_2^* - g_2.$$

Now from the mean value theorem for integrals there will exist two numbers y_2^* and y_2^{**} with $f < y_2^* < \bar{y}_2$, and $\bar{y}_2 < y_2^{**} < h$ such that

$$\operatorname{cov}(U'(c_2),\pi) = -P_1 k\overline{(1/M_2^s)} \int_d^e \left\{ U'(c_2^*) \int_f^{\bar{y}_2} (y_2 - \bar{y}_2)\phi(y_2)\mathrm{d}y_2 \right.$$

$$\left. + U'(c_2^{**}) \int_{\bar{y}_2}^h (y_2 - \bar{y}_2)\phi(y_2)\mathrm{d}y_2 \right\} \gamma(g_2)\mathrm{d}g_2$$

Since $U'(c_2^*) > U'(c_2^{**})$ for any given value g, the term in braces must always be non-positive. Thus,

$$\operatorname{cov}(U'(c_2),\pi) > 0.$$

Appendix B

This appendix is presented to provide the interested reader a taste of the technical aspects of some of the comparative static results discussed in the text. The results of those comparative static exercises not discussed here can be easily deduced by mimicking the line of argument utilized below. First, the impact on P_1 and c_1 from changes in $\mathrm{E}[M_2^s]$ and $\mathrm{E}[y_2]$ due to shifts in μ_2 and $\bar{y}_2$ can be uncovered by taking the total differential of eqs. (17) and (18) while taking note of the information provided by (19). The resulting system of two equations that one gets is

$$\begin{bmatrix} U''(c_1)+\beta(1+r^*)^2\mathrm{E}[U''(c_2)] & -\beta(1+r^*)^2v'(1)(M_1^s/P_1^2)\mathrm{E}[U''(c_2)] \\ -(1+r^*)\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)] & (1+r^*)v'(1)(M_1^s/P_1^2)\mathrm{E}[U''(c_1)(P_1/P_2(1+r^*)-v'(1)-1)] \\ & +\mathrm{E}[U'(c_2)(1/P_2(1+r^*)+v''(1)M_1^s/P_1^2y_1)] \end{bmatrix}$$

$$\begin{bmatrix} \mathrm{d}c \\ \mathrm{d}P_1 \end{bmatrix} = \begin{bmatrix} \beta(1+r^*)\mathrm{E}[U''(c_2)](1-v(k))\mathrm{d}\bar{y}_2 \\ -E[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)](1-v(k))\mathrm{d}\bar{y}_2 \\ -\mathrm{E}[U'(c_2)kP_1/M_2^s(1+r^*)]\mathrm{d}\bar{y}_2+\mathrm{E}[U'(c_2)P_1ky_2/M_2^s(1+r^*)]\mathrm{d}\mu_2 \end{bmatrix} \tag{B.1}$$

[note $v'(1)\equiv v'(M_1^s/P_1y_1)$],

where Δ is the determinant of the two-by-two matrix on the left-hand side of (B.1)

$$\Delta = U''(c_1)(1+r^*)v'(1)(M_1^s/P_1^2)\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)]$$
$$+\{U''(c_1)+\beta(1+r^*)^2\mathrm{E}[U''(c_2)]\}\{\mathrm{E}[U'(c_2)$$
$$\times(1/P_2(1+r^*)+v''(1)M_1^s/P_1^2y_1)]\}.$$

Solving the preceding system of equations, (B.1) yields

$$\partial c_1/\partial\bar{y}_2 = \{\beta(1+r^*)\mathrm{E}[U''(c_2)](1-v(k))\mathrm{E}[U'(c_2)(1/P_2(1+r^*)$$
$$+v''(1)M_1^s/P_1^2y_1)]-\beta(1+r^*)^2v'(1)(M_1^s/P_1^2)\mathrm{E}[U''(c_2)]\mathrm{E}[U'(c_2)$$
$$\times kP_1/M_2^s(1+r^*)]\}/\Delta>0, \tag{B.2}$$

$$\partial P_1/\partial\bar{y}_2 = \{-U''(c_1)\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)](1-v(k))$$
$$-[U''(c_1)+\beta(1+r^*)^2\mathrm{E}[U''(c_2)]]\mathrm{E}[U'(c_2)$$
$$\times kP_1/M_2^s(1+r^*)]\}/\Delta<0, \tag{B.3}$$

$$\partial c_1/\partial\mu_2 = \{\beta(1+r^*)^2v'(1)(M_1^s/P_1)\mathrm{E}[U''(c_2)]$$
$$\times\mathrm{E}[U'(c_2)ky_2/M_2^s(1+r^*)]\}/\Delta<0, \tag{B.4}$$

$$\partial P_1/\partial\mu_2 = \{(U''(c_1)+\beta(1+r^*)^2\mathrm{E}[U''(c_2)])$$

$$\times\,\mathrm{E}[U'(c_2)P_1 k y_2/M_2^s(1+r^*)]\}/\Delta > 0. \tag{B.5}$$

The determinant, Δ, can be seen to be comprised of two terms of which the second is unambiguously negative. The first term is non-negative because decreasing absolute risk aversion implies that the expression $\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)]$ is positive (see appendix C). This expression reflects changes in the risk premium due to changes in the marginal utility of second period consumption which occurs, for example, when any of y_1, g_1, P_1, $\bar{y}_2$, or $\bar{g}_2$ shift. Since the first term, which equals zero when the model is deterministic, is likely to be small in value, Δ can be safely taken to be negative in value (see appendix C for further details). Now, it can be easily deduced that the derivatives given in (B.2) to (B.5) take the signs shown.

The other comparative static results given in table 1 can be obtained by using the same line of argument. It is easy to see that the two-by-two matrix on the left-hand side of (B.1) remains the same in all of these exercises and all that changes is the two-by-one displacement vector on the right-hand side of this equation. For instance, performing the above exercise for a temporary change in y_1, the results obtained would be

$$\partial P_1/\partial y_1 = -(1+r^*)(1-v(1))U''(c_1)\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)$$

$$-v'(1)-1)]/\Delta - v''(1)(M_1/P_1 y_1^2)\mathrm{E}[U'(c_2)]\{U''(c_1)$$

$$+\beta(1+r^*)^2\mathrm{E}[U''(c_2)]\}/\Delta - (1+r^*)v'(1)(M_1^s/P_1 y_1)U''(c_1)$$

$$\times\,\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*)-v'(1)-1)]/\Delta < 0, \tag{B.6}$$

$$0 < \partial c_1/\partial y_1 = (1-v(1))\{\beta(1+r^*)^2\mathrm{E}[U''(c_2)]\mathrm{E}[U'(c_2)(1/P_2(1+r^*)$$

$$+v''(1)M_1^s/P_1^2 y_1)]/\Delta\}$$

$$+\beta(1+r^*)^2 v'(1)(M_1^s/P_1 y_1)\mathrm{E}[U''(c_2)]$$

$$\times\,\mathrm{E}[U'(c_2)/P_2(1+r^*)]/\Delta < (1-v(1)). \tag{B.7}$$

In the expression for $\partial P_1/\partial y_1$ it is easily discerned that the first two terms are negative while the last is positive. However, it can be shown that $(1-v(1))$ is larger in magnitude than $v'(1)(M_1^s/P_1 y_1)$ so that the sum of the first and third terms will be negative, and consequently so will $\partial P_1/\partial y_1$.[16] Now, examine the expression for $\partial c_1/\partial y_1$. The first term is positive while the second is negative. It turns out that the sum of these two terms is positive since $(1-v(1))$ is greater than $v'(1)(M_1^s/P_1y_1)$. The fact that $\partial c_1/\partial y_1 < (1-v(1))$ follows from the definition of Δ.

Another comparative static exercise of interest is the impact of a change in the expected level of future government spending on today's price level. After carrying out the necessary mathematics, one gets

$$\partial P_1/\partial \bar{g}_2 = U''(c_1)\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*) - v'(1) - 1)]/\Delta > 0.$$

A change in expected future government expenditure only affects today's price level insofar as it alters the risk premium on holding money. It has been argued that the numerator of the above expression is likely to be small, so that this channel of effect on the price level should be thought of as being of secondary importance. This example is of theoretical interest though, even if it isn't likely to be quantitative significance, because it shows how the risk premium term operates in the model.

Lastly, a brief discussion will be undertaken of the impact of a permanent change in y_1 on P_1 and c_1. This is by far the trickiest of the comparative static exercises undertaken. As a benchmark from which to undertake this exercise, let $1/\beta = (1+r^*)$, and assume that $v(1) = v(k)$. To begin with, note that the system is linear in its displacements. Thus, the impact effect of a permanent change in today's price level, P_1, is simply the sum of $\partial P_1/\partial y_1$ and $\partial P_1/\partial \bar{y}_2$ as given by (B.3) and (B.6). Since both of these expressions are negative the sum must also be negative. Consequently, a permanent change in income unambiguously leads to a fall in today's price level. But more than this can be said. Note that in the deterministic case of the model, all terms involving $\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*) - v'(1) - 1)]$ will be identically zero. Then, it immediately follows in the deterministic version of the model that for a permanent change in current income

$$\partial P_1/\partial y_1 = P_1/y_1.$$

In the uncertain case of the model with small income risks, the above result will hold as an approximation. By adding (B.2) and (B.7) the impact of a

[16]To see this, suppose that $(1-v(M_1^s/P_1y_1))=0$, when $M_1^s/P_1y_1=0$. That is, if no money is held, all of income is absorbed in transactions costs. Then, by taking a first order Taylor expansion of $(1-v(M_1^s/P_1y_1))$ around zero, one gets

$$(1-v(M^{s1}/P_1y_1)) = -v'(r)M_1^s/P_1y_1 \quad \text{with} \quad 0<r<M_1^s/P_1y_1.$$

Now, since $-v'(\cdot)$ is decreasing in M_1^s/P_1y_1, it follows that

$$(1-v(M_1^s/P_1y_1)) > -v'(M_1^s/P_1y_1)M_1^s/P_1y_1.$$

If $(1-v(M_1^s/P_1y_1))>0$, when $M_1^s/P_1y_1=0$, the proof goes through a fortiori.

permanent increase in income on current consumption can be obtained. Since both of the terms are positive their net effect must be positive. When the model is deterministic, one obtains the following result:

$$\partial c_1/\partial y_1 = (1 - v(1)).$$

Again, for small income risks, the result will hold approximately.[17]

Appendix C[18]

Recall from (13) that

$$\beta \mathrm{E}[U'(c_2)(1/P_2 - v'(1)(1+r^*)/P_1 - (1+r^*)/P_1)] = 0.$$

Now, define Z as

$$Z = P_1/P_2(1+r^*) - v'(1) - 1.$$

Using expression (11) the above equation can be rewritten as

$$Z = P_1 k y_2/M_2^s(1+r^*) - v'(1) - 1.$$

Using the first expression for Z, it can be seen that

$$\mathrm{E}[U''(c_2)Z] = 0.$$

Now, let $\bar{y}_2$ be the value of y_2 that sets Z equal to zero, and then make the following definition of $\bar{c}_2$:

$$\bar{c}_2 = (1 - v(1))y_1(1+r^*) + (1 - v(k))\bar{y}_2 - (c_1 + g_1)(1+r^*) - g_2.$$

Assume that $Z > 0$ because $y_2 > \bar{y}_2$. Decreasing absolute risk aversion would imply that

$$-U''(c_2)/U'(c_2) < -U''(\bar{c}_2)/U'(\bar{c}_2), \quad \text{therefore}$$

$$U''(c_2)Z > (U''(\bar{c}_2))U'(\bar{c}_2)Z. \tag{A}$$

Alternately, assume that $Z < 0$ because $y_2 < \bar{y}_2$. Decreasing absolute risk

[17] The results in appendix C can be used to show formally that the last two expressions hold approximately for small income risks.

[18] Sandmo (1968).

aversion would imply that

$$-U''(c_2)/U'(c_2) > -U''(\bar{c}_2)/U'(\bar{c}_2), \quad \text{therefore}$$

$$U''(c_2)Z > (U''(\bar{c}_2)/U'(\bar{c}_2))U'(c_2)Z. \tag{B}$$

It follows from (A) and (B) that for all Z

$$U''(c_2)Z > (U''(\bar{c}_2)/U'(\bar{c}_2))U'(c_2)Z.$$

Taking expectations of both sides of the above equation yields

$$\mathrm{E}[U''(c_2)Z] > \mathrm{E}[(U''(\bar{c}_2)/U'(\bar{c}_2))U'(c_2)Z] = 0$$

$$\Rightarrow \mathrm{E}[U''(c_2)(1/P_2 - v'(1)(1+r^*)/P_1 - (1+r^*)/P_1)] > 0$$

since $U''(\bar{c}_2)/U'(\bar{c}_2)$ is a constant and $\mathrm{E}[U'(c_2)Z] = 0$ from (13).

Now, for small income risks it seems that $\mathrm{E}[U''(c_2)Z]$ should be approximately equal to zero in value. Through the use of (14), this expression can be rewritten as

$$\mathrm{E}[U''(c_2)Z] = \mathrm{cov}(U'(c_2), \pi)\mathrm{E}[U''(c_2)]/(1+r^*)\mathrm{E}[U'(c_2)]$$
$$- \mathrm{cov}(U''(c_2), \pi)/(1+r^*) > 0.$$

Note that results of appendix A imply that the first term is unambiguously negative, and hence that the second term which is being subtracted must be negative and dominant in magnitude. That is, the absolute value of the first term is bounded in magnitude by the absolute value of the second. Thus, as the second term approaches zero in magnitude, so must the first. Taking a first order approximation of the second covariance term around $\bar{y}_2$, while adopting the formulae and notation from appendix A, yields

$$\mathrm{cov}(U''(c_2), \pi) \simeq -P_1 k(\overline{1/M_2^s})(1 - v(k))\sigma_y^2 \int U'''(\tilde{c}_2)\gamma(g_2)\mathrm{d}g_2 < 0,$$

where $\tilde{c}_2$ is defined as the second period consumption associated with the income level $\bar{y}_2$. The variable $\tilde{c}_2$ is still random as g_2 is still random. Obviously, as $\sigma_y^2 \to 0$ so does the covariance term in question. Consequently, for small risks in income it is the case that

$$\mathrm{E}[U''(c_2)(P_1/P_2(1+r^*) - v'(1) - 1)] \simeq 0.$$

In the deterministic case of the model, it can quickly be discerned from

(14) that $-v'(1)=(\pi+r^*)/(1+r^*)$. In the uncertain setting it is hard to believe that income would be subjected to such extreme variations so as to induce large movements in the inflation rate. Hence, for most realizations of y_2, one would think that $P_2/P_1(1+r^*)-v'(1)-1$ is close to zero in value. Consequently, for the above reasons, $\mathrm{E}[U''(c_2)(P_2/P_1(1+r^*)-v'(1)-1)]$ should be a small number and as a result Δ should be negative.

References

Aschauer, David and Jeremy Greenwood, 1983, A further exploration in the theory of exchange rate regimes, Journal of Political Economy 91, no. 5, Oct., 868–875.

Barro, Robert J., 1981, Output effects of government purchases, Journal of Political Economy 89, no. 6, Dec., 1086–1121.

Bilson, John F. O., 1978, Rational expectations and the exchange rate, in: Jacob A. Frenkel and Harry G. Johnson, eds., The economics of exchange rates (Addison-Wesley, London).

Dornbusch, Rudiger and Stanley Fischer, 1980, Exchange rates and the current account, American Economic Review 70, no. 5, Dec., 960–971.

Dornbusch, Rudiger and Jacob Frenkel, 1973, Inflation and growth: Alternative approaches, Journal of Money, Credit and Banking 5, no. 1, Part I, Feb., 141–156.

Fama, Eugene F. and André Farber, 1979, Money, bonds, and foreign exchange, American Economic Review 79, no. 4, Sept., 639–649.

Flavin, Marjorie A., 1981, The adjustment of consumption to changing expectations about future income, Journal of Political Economy 89, no. 5, Oct., 974–1009.

Greenwood, Jeremy, 1983, Relative prices, the trade balance and the balance of payments, Research Report no. 8306, Jan. (University of Western Ontario, Department of Economics, London, Ont.).

Hall, Robert E., 1978, Stochastic implication of the life cycle-permanent income hypothesis: Theory and evidence, Journal of Political Economy 86, no. 6, Dec., 971–987.

Helpman, Elhanan, 1981. An exploration in the theory of exchange rate regimes, Journal of Political Economy 89, no. 5, Oct., 865–890.

Johnson, Harry G., 1973, The monetary approach to the balance of payments, in: Further essays in monetary economics (Allen and Unwin, London).

Jones, Ronald W. and Douglas D. Purvis, 1981, International differences in response to common external shocks: The role of purchasing power parity, unpublished (University of Rochester, Rochester, NY).

Kouri, Pentti J., 1981, The effects of risk on interest rates: A synthesis of the macroeconomic and financial views, NBER working paper series, no. 643, March.

Rodriguez, Carlos A., 1980, The role of trade flows in exchange rate determination: A rational expectations approach, Journal of Political Economy 88, no. 6, Dec., 1148–1158.

Sandmo, Agnar, 1968, Portfolio choice in a theory of saving, Swedish Journal of Economics LXX, no. 2, 106–122.

Sargent, Thomas J., 1978, Rational expectations, econometric exogeneity, and consumption, Journal of Political Economy 86, no. 4, Aug., 673–700.

Stockman, Alan C., 1980, A theory of exchange rate determination, Journal of Political Economy 88, no. 4, Aug., 673–698.

Stockman, Alan C., 1983, Real exchange rates under alternative nominal exchange rate systems, Journal of International Money and Finance, forthcoming.