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Efficient Investment in Children[1]

S. Rao Aiyagari and Jeremy Greenwood

Department of Economics, University of Rochester, Rochester, New York 14627-0156
gree@troi.cc.rochester.edu

and

Ananth Seshadri

Department of Economics, University of Wisconsin, Madison, Wisconsin 53706-1320
aseshadr@ssc.wisc.edu

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Many would say that children are society's most precious resource. So, how should we invest in them? To gain insight into this question, a dynamic general equilibrium model is developed where children differ by ability. Parents invest time and money in their offspring, depending on their altruism. This allows their children to grow up as more productive adults. First, the efficient allocation is characterized. Next, this is compared with the outcome that arises when financial markets are incomplete. The situation where child-care markets are also lacking is then examined. Additionally, the consequences of impure altruism are analyzed. *Journal of Economic Literature* Classification Numbers: D1, D31, D58, I2.

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1. INTRODUCTION

In the U.S. economy a male in the top 5th percentile earns about 8.6 times the labor income of one in the bottom 5th. The correlation between a father's and a son's earnings is high, too, somewhere between 0.40 and 0.65. Many take this as *prima facie* evidence that markets fail. They believe that differences in ability cannot be so great as to explain such great

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differences in income. They also feel that the transmission of genetic factors across generations cannot be so high as to explain this low degree of intergenerational mobility. This may be true.

The problem for the economist is that ability is not well observed. Further, psychometric testing provides ordinal, and not cardinal, measures of ability. Consequently, they say little about the dispersion in ability.[2] Also, there is evidence suggesting that testing is influenced by family background, factors such as whether one's parents went through a divorce early in life.[3] Family background in turn is related to family income. Furthermore, even if a true measure of ability could be found you would need to know how ability translates into earnings, ceteris paribus. This translation will depend on the economy's production technologies, at a minimum. Should a person with twice the ability of another earn four times as much or twice as much? Who knows?

An obvious factor influencing earnings may be investments by parents in the human capital of their children. Poor parents have less wherewithal to invest in their children than do rich ones. They also can't borrow against their offspring's income in order to finance their kid's human capital formation. This will lead to parental background being an important determinant in income, besides ability.[4] It will lead to persistence in income across the generations of a dynasty. There may be public programs that can alleviate such imperfections. It is important to invest in children, however, while they are still young. To quote Heckman [7, p. 96]:

> The reason is this: Cognitive ability is formed relatively early in life and becomes less malleable as children age. By age 14, basic cognitive abilities seem to be fairly well set. Since ability promotes academic progress, successful interventions early in the life cycle of learning lead to higher overall achievement. By the time individuals finish high school, scholastic ability is determined, and tuition policy will have little effect on college attendance.

And, Currie and Thomas [5] find that school test scores at the age of 7 are significant determinants of future labor market outcomes. Therefore, the focus of the current analysis is on children where such market imperfections are likely to weigh the heaviest, as opposed to young adults.

[2] For example, IQ test scores are normalized to have a mean of 100 and a standard deviation of 15. Hence, dispersion in IQ cannot be used to measure dispersion in ability.

[3] See Heckman, Hsse and Rubinstein [8]. Also, Neal and Johnson [17] find that AFQT scores are influenced by family background and school environments.

[4] "(T)he disadvantages young black workers now face in the labor market arise mostly from the obstacles they faced as children in acquiring productive human capital", say Neal and Johnson [17, p. 871].

So, what determines parental investment in children? The answer to this question will depend upon how the world is viewed. To gain some insight into this issue, an overlapping generations model is constructed where children differ by ability. Ability has a random component to it. In line with the classic papers by Becker and Tomes [3] and Loury [14], the productivity of an adult is determined by his ability and the amount of human capital investment that his parents undertook when he was a child. The amount that a parent invests in a child depends on how altruistic he is towards his child, as well as upon the assumed structure of markets.

Several different market structures are analyzed. To begin with, the efficient equilibrium is modelled. Then, a world with incomplete financial markets is entertained. There are two sources of incompleteness. First, parents are unable to purchase insurance on the ability of their grandchildren. Second, they face borrowing constraints when educating their children. Specifically, a parent cannot pass on any debts to his offspring. The analysis here has the flavor of Aiyagari [1] and Laitner [12], who analyze the behavior of savings in an economy with incomplete financial markets and idiosyncratic risk. While the focus is different, the work here is also related to Knowles's [11] study of the implications of the Becker and Barro [2] fertility model, where parents decide upon both the quality and quantity of children, for modeling the distribution of income. Next, the lack of child-care markets is introduced into the environment with incomplete financial markets. In this situation a parent must use his own time to improve the human capital of his child.

The implications of these varying structures on efficiency, output, and the distribution of income are catalogued. In the general equilibrium model developed here, the absence of insurance markets and the presence of borrowing constraints does not necessarily lead to underinvestment in children. It can lead to overinvestment. The investment is inefficient, however, in the sense that it is not directed toward the children who warrant it the most. Impure altruism towards children has a big impact on investment in children. This may be troubling for economists. The fact that tastes are interdependent, in the sense that a child's welfare enters a parent's utility function, does not imply that an equilibrium lacks Pareto optimality.[5] How a parent should love his offspring takes one outside of the realm of economics. Tastes may evolve over time, though, since a little over one hundred years ago children had a capital asset aspect associated with them; they were expected to work when young and to provide old-age support to their parents when grown up.

[5] So long as the child's welfare does not enter into someone else's utility there will be no incentive for one benefactor to free ride off another.

2. ENVIRONMENT

Generational structure. The environment is a discrete-time infinite-horizon economy with periods denoted by $t \in \{0, 1, 2, 3, \ldots\}$. In each period there is a continuum of children. Adults live for two periods. In the first period of life they are young, while in the second period they are old. At the end of each period t the old adults die. They are replaced by a new generation of children spawned by the period-t generation of young adults. These children will become young adults in period $t+1$ who will then have their own children. Life goes on in the future in similar fashion.

Ability and productivity. Children are distributed according to innate ability, a. A child's ability may be a function of his parent's ability, a_{-1}, in line with the cumulative distribution function, $A(a \mid a_{-1})$. The distribution function A is taken as a primitive. The ability of a child is perfectly known in period t. The initial distribution for abilities will be given by $\mathbf{A}_0 = \mathbf{A}$ where $\mathbf{A}(x) = \int A(x \mid a)\, d\mathbf{A}(a)$. That is, initial abilities are drawn from the stationary distribution associated with A with the implication that the cross-sectional distribution of ability will be given by $\mathbf{A}_t = \mathbf{A}$ for all t.

Adults differ according to their productivities, π. Parents can influence the productivity of their offspring by investing time and money in them. There is a fixed cost ϕ associated with educating a child. Now, consider a young parent who invests m units of resources (in addition to the fixed cost ϕ) and n units of child-care time in his child. The child will grow up next period with productivity, π', as described by

$$\pi' = H(a, m, n), \tag{1}$$

where $H(a, m, n) = a$ if either $n = 0$ or $m = 0$. The function H is taken as a primitive. Assume that H is strictly increasing in all its arguments and that H_{12}, H_{13}, and $H_{23} > 0$. Furthermore, suppose that H is strictly concave in m and n, both jointly and separately. When resources are invested in a child he will be labeled as skilled. Otherwise, he will be called unskilled. The above assumptions guarantee that an efficient allocation will dictate that the amount of money and time invested in a child will increase with ability.

Goods production. Each young adult has one unit of time. He can spend his time either manufacturing goods or supplying services on a child-care market. If a young adult spends one unit of time making goods then he can supply π efficiency units of labor in production. Suppose that this adult had drawn the ability level a_{-1} last period as a child. A unit of time in child-care then generates a_{-1} efficiency units of labor in this activity. Skilled agents have a comparative advantage in manufacturing goods since for them $\pi > a_{-1}$ while for unskilled agents $\pi = a_{-1}$. Old adults can't work.

Output, $\mathbf{o}$, is produced according to the constant-returns-to-scale production function

$$\mathbf{o} = O(\mathbf{k}, \mathbf{l}),$$

where $\mathbf{k}$ and $\mathbf{l}$ are the aggregate quantities of capital and labor used in production. Aggregate labor is the sum over the efficiency units of effort supplied by individuals to manufacturing.

Output can be used for consumption, $\mathbf{c}$, investment in capital goods, $\mathbf{i}$, and investment in children, $\mathbf{m}$. In other words

$$\mathbf{c} + \mathbf{i} + \mathbf{m} = \mathbf{o}.$$

Capital goods accumulate according to the law of motion

$$\mathbf{k}' = (1-\delta)\,\mathbf{k} + \mathbf{i}. \tag{2}$$

3. EFFICIENCY

What will an efficient markets equilibrium look like? To answer this question some notation will be introduced. Let $\Pi_t(\pi)$ denote the distribution of adults according to productivity in period t. The initial distribution Π_0 is predetermined, while future Π's will be determined endogenously in a manner discussed below. The amount of time that a young adult of productivity π spends in production in period t will be represented by $L_t(\pi)$. In similar fashion, $M_t(a)$ will specify the amount of goods invested (excluding the fixed cost ϕ) in period t on a child of ability a. Likewise $N_t(a)$ will denote the quantity of young adult time spent in period t on a type-a child. Note that in an efficient markets equilibrium, investment in a child will depend solely on the child's ability and nothing else, such as the ability of his or her parents.

Given this notation, the productivity distributions evolve as

$$\Pi_{t+1}(\pi) = m\{a\colon H(a, M_t(a), N_t(a)) \leqslant \pi\}, \tag{3}$$

where m is the measure on the set of abilities corresponding to the stationary distribution, $\mathbf{A}$.[6] Let

$$\mathcal{S}_t = \{a\colon H(a, M_t(a), N_t(a)) > a\},$$

[6] Let $\pi' = H(a, N(a), M(a)) \equiv G(a)$. Now, suppose that G has a continuously differentiable inverse and that A has a continuous density, A_1. Then, $\Pi_1(\pi') = A_1(G^{-1}(\pi'))\,|G_1^{-1}(\pi')|$.

so that $\mathcal{S}_t$ represents the set of children in period t that become skilled in $t+1$. The set of unskilled children, $\mathcal{U}_t$, will be given by the complement of this set so that $\mathcal{U}_t = \mathcal{S}_t^c$.

The amount of goods invested in children is given by $\mathbf{m} = \int_{\mathcal{S}} [M(a)+\phi]\, d\mathbf{A}(a)$, so the resource constraint for this economy reads

$$\mathbf{c}+\mathbf{i}+\int_{\mathcal{S}} [M(a)+\phi]\, d\mathbf{A}(a) \leqslant O(\mathbf{k}, \mathbf{l}). \tag{4}$$

Finally, the amount of labor that is used in production (measured in efficiency units) must be less than the total supply of it minus the amount that is used in child-care so that

$$\mathbf{l} = \int \pi \, d\Pi(\pi) - \int_{\mathcal{S}} N(a)\, d\mathbf{A}(a).$$

Assuming that only unskilled agents work in the child-care sector (discussed in the next section), the amount demanded for child-care in any period t must be less than the supply of unskilled agents so that

$$\int_{\mathcal{S}_t} N_t(a)\, d\mathbf{A}(a) \leqslant \int_{\mathcal{U}_{t-1}} a\, d\mathbf{A}(a)\,. \tag{5}$$

3.1. *Characterizing Efficient Allocations*

Efficiency means that it is not possible to have more consumption at some date without having less consumption at some other, assuming that leisure is not valued. The problem of efficient investment in children is to determine the schedules $L_t(\pi)$, $M_t(a)$ and $N_t(a)$ in each period given this efficiency criterion.

Characterizing the schedule L_t is straightforward. Assume that there are a sufficient number of unskilled agents to meet the economy's child-care requirements. It's obvious that there should be some π_t^* such that

$$\begin{aligned} L_t(\pi_t) &= 1, \qquad \text{if} \quad \pi_t \geqslant \pi_t^*, \\ L_t(\pi_t) &\leqslant 1, \qquad \text{if} \quad \pi_t \leqslant \pi_t^*. \end{aligned} \tag{6}$$

This follows because skilled agents have a comparative advantage in goods production; that is, their productivity in goods production, π_t, exceeds their productivity in child-care, a_{t-1}. Now, since $\pi_{t+1} = H(a_t, M_t(a_t), N_t(a_t))$, it transpires that any cutoff rule for π_{t+1}, or π_{t+1}^* will amount to a cutoff rule for a_t, or a_t^*.

In light of the above, rewrite (4) as

$$\mathbf{c}_t + \mathbf{i}_t + \int_{a_t^*} [M_t(a) + \phi]\, dA(a) \leqslant O\left(\mathbf{k}_t, \int \pi\, d\Pi_t(\pi) - \int_{a_t^*} N_t(a)\, d\mathbf{A}(a)\right). \quad (7)$$

Observe that output next period, $\mathbf{o}_{t+1}$, can be rewritten to obtain

$$\mathbf{o}_{t+1} = O\left(\mathbf{k}_{t+1}, \int_{a_t^*} H(a, M_t(a), N_t(a))\, d\mathbf{A}(a) + \int^{a_t^*} a d\mathbf{A}(a) - \int_{a_{t+1}^*} N_{t+1}(a)\, d\mathbf{A}(a)\right).$$

The planning problem. Let $\{p_t\}_{t \geqslant 0}$ be a sequence of "efficiency prices" with $p_t > 0$ for all t. Then any allocation which maximizes $\sum_{t \geqslant 0} p_t \mathbf{c}_t$ is efficient. Interpret $p_t / p_{t+1} = r_t$ as the *gross* interest rate from t to $t+1$. The primary interest here is in steady states. Without loss of generality, look at the problem of maximizing $(p_t \mathbf{c}_t + p_{t+1} \mathbf{c}_{t+1})$ with respect to a_{t+1}^*, $M_t(a)$, $N_t(a)$ and $\mathbf{k}_{t+1}$ and then look at the steady-state versions of the first-order necessary conditions characterizing the solution.

Therefore, the problem of efficient investment in children is to maximize

$$p_t \left\{O\left(\mathbf{k}_t, \int \pi\, d\Pi_t(\pi) - \int_{a_t^*} N_t(a)\, d\mathbf{A}(a)\right) - \mathbf{i}_t - \int_{a_t^*} [M_t(a) + \phi]\, d\mathbf{A}(a)\right\}$$
$$+ p_{t+1} \left\{O\left(\mathbf{k}_{t+1}, \int_{a_t^*} H(a, M_t(a), N_t(a))\, d\mathbf{A}(a) + \int^{a_t^*} a dA(a) - \int_{a_{t+1}^*} N_{t+1}(a)\, d\mathbf{A}(a)\right) - \mathbf{i}_{t+1} - \int_{a_{t+1}^*} [M_{t+1}(a) + \phi]\, d\mathbf{A}(a)\right\}.$$

subject to (2). The first-order necessary conditions associated with the above problem are:

$$a_t^*\colon p_t[O_2(\cdot t)\, N_t(a_t^*) + M_t(a_t^*) + \phi] - p_{t+1} O_2(\cdot t+1)[H(\cdot t+1) - a_t^*] = 0, \quad (8)$$

$$M_t(a)\colon -p_t + p_{t+1} O_2(\cdot t+1)\, H_2(\cdot t+1) = 0 \qquad (\text{for } M_t(a) > 0), \quad (9)$$

$$N_t(a)\colon -p_t O_2(\cdot t) + p_{t+1} O_2(\cdot t+1)\, H_3(\cdot t+1) = 0 \qquad (\text{for } N_t(a) > 0), \quad (10)$$

$$\mathbf{k}_{t+1}\colon p_t = p_{t+1}[O_1(\cdot t+1) + (1 - \delta)]. \quad (11)$$

The notation $X(\cdot t)$ signifies that the function X is being evaluated at its date-t arguments.

The steady state is characterized by the following equations:

$$M(a^*)+\phi+N(a^*)\,w=\frac{wH(a^*,M(a^*),N(a^*))}{r}-\frac{wa^*}{r}, \tag{12}$$

$$wH_2(a,M(a),N(a))=r \qquad (\text{for } M(a)>0), \tag{13}$$

$$H_3(a,M(a),N(a))=r \qquad (\text{for } N(a)>0), \tag{14}$$

$$r=O_1(\cdot)+(1-\delta). \tag{15}$$

Here

$$w=O_2(\cdot) \tag{16}$$

represents the wage rate for an efficiency unit of labor. Equation (12) states that the cost of becoming skilled, $M(a^*)+\phi+N(a^*)\,w$, should equal the benefit or the discounted skill premium, $wH(a^*, M(a^*), N(a^*))/r-wa^*/r$, at the cutoff level of ability, a^*. Again note that equation (12) can equivalently be thought of as defining a cutoff rule for productivity, π^*, which is defined by $\pi^*=H(a^*, M(a^*), N(a^*))$. Next, society should invest time in a child up until the point where the discounted marginal return $wH_3(a, M(a), N(a))/r$ equals the cost of the extra child-care, w. This is what (14) states. Condition (13) states a similar condition for resources. Last, (15) is a standard condition equating the marginal product of capital to the interest rate.

The solution has the following feature. As noted, skilled adults spend all their time producing. Some unskilled adults will devote their time to producing, while others will spend it taking care of children. Some children will have positive amounts of adult time and goods invested in them and will (when they become young adults) work full time in production as skilled agents. The rest of the children will have zero adult time and goods invested in them and will (when they become young adults) work as unskilled agents either in production or taking care of the next generation of children. Basically, the above conclusion is a result of the assumption that skilled agents have a comparative advantage in producing manufacturing goods.

4. MARKET ARRANGEMENTS

The setting. Can the efficient allocation be supported as a competitive equilibrium? To answer this question, something has to be said about preferences. Assume that adults are matched one-to-one with children and that each adult cares about his child altruistically. Each young parent has preferences of the form

$$U(c^y)+\beta E[U(c^{o'})+\theta V'], \qquad 0<\beta<1, \qquad 0<\theta\leqslant 1, \tag{17}$$

where c^y and $c^{o'}$ are his consumptions when young and old. Here V' denotes the expected lifetime utility that his child will realize upon growing up. The young adult attaches the weight θ to his offspring's expected lifetime utility and he discounts the future at rate β.[7] The analysis presumes that children cannot transact for themselves. Hence, there would be no investment in a child if it was not for his parent's altruism (i.e., if it wasn't for the fact that $\theta>0$). The case where $\theta<1$ will be labeled "impure" altruism.[8]

There are one-period-ahead complete insurance markets so that an adult can insure against the ability level of his grandchild next period. Since the focus of the analysis is on steady states, all prices will be assumed to be constant over time. Let $q(a' \mid a)$ denote the price of a claim which delivers one unit of consumption next period if the grandchild's ability level is a' and nothing otherwise, conditional on the young adult having a child of ability a. The quantity of such claims that the young adult purchases is $s(a' \mid a)$. Last, the young adult can leave, when old, a bequest to his offspring, if he desires. In particular, if he wants his offspring to receive b' units of consumption in a bequest then he will have to put aside b'/r units of consumption when old, where r is the market rate of interest on a one-period bond. This bequest can be negative.

Choice problems. The dynamic-programming problem facing a young parent can now be written as

$$V(\pi, a, b) = \max_{s(a' \mid a),\, m,\, n} \left\{ U(c^y)+\beta \int J(\pi', a', s(a' \mid a)+b)\, A_1(a' \mid a)\, da' \right\}, \tag{18}$$

[7] These are similar to the preferences considered in a classic paper by Phelps and Pollak [19]. Each generation assigns a more primal role to its own utility vis à vis its offspring's. These preferences are non-stationary, however, since the next generation will assign a primal role to its own utility. As Phelps and Pollak [19] note, Frank P. Ramsey termed the practice of discounting the next generation's utility "ethically indefensible."

[8] The word impure arises from Edgeworth [6, p. 16] who said "(f)or between the two extremes of Pure Egoistic and Pure Universalistic there may be an indefinite number of impure methods; wherein the happiness of others as compared by the agent (in a calm moment) with his own, neither counts for nothing, not (*sic*) yet 'counts for one,' but *counts for a fraction*."

subject to (1) and

$$c^y + m + \phi I(\pi', a) + wn + \int q(a' \mid a)\, s(a' \mid a)\, da' = w\pi, \tag{19}$$

where the indicator function I is defined so that

$$I(\pi', a) = \begin{cases} 1, & \pi' > a, \\ 0, & \text{otherwise.} \end{cases}$$

Here

$$J(\pi', a', s(a' \mid a) + b) = \max_{b'} \{U(c^{o'}) + \theta V(\pi', a', b')\}, \tag{20}$$

subject to

$$c^{o'} + b'/r = s(a' \mid a) + b. \tag{21}$$

When the agent is old he will have a wealth level of $s(a' \mid a) + b$ and a grandchild of ability a'. At this time the agent will have to decide how much to leave to his adult child in bequests or b'. Problem (20) describes the decision making at this stage of life. Therefore, $J(\cdot)$ is the indirect utility function for an old adult.[9, 10] The first-order necessary conditions associated with this problem are:

$$s(a' \mid a)\colon U_1(c^y)\, q(a' \mid a) = \beta J_3(\pi', a', s(a' \mid a) + b)\, A_1(a' \mid a), \tag{22}$$

$$m\colon U_1(c^y) = \beta H_2(a, m, n) \int J_1(\pi', a', s(a' \mid a) + b)\, A_1(a' \mid a)\, da' \quad \text{(when } m > 0\text{)}, \tag{23}$$

$$n\colon U_1(c^y)\, w = \beta H_3(a, m, n) \int J_1(\pi', a', s(a' \mid a) + b)\, A_1(a' \mid a)\, da' \quad \text{(when } n > 0\text{)}, \tag{24}$$

[9] Observe that each parent assumes that his offspring will do what is in the descendent's best interest. That is, while the parent doesn't assign a primal role to the offspring's utility he correctly assumes that his offspring will. The resulting equilibrium is time consistent.

[10] The forms of problems (18) and (20) would become more complicated if children overlapped more periods with their parents, and/or if children also cared about their parents. Strategic considerations between parents and children would then emerge. See Laitner [13] for an excellent review of this literature.

and

$$b': U_1(c^{o'})/r = \theta V_3(\pi', a', b'). \tag{25}$$

Last, an application of the Benveniste and Scheinkman and envelope theorems to (18) and (20) yields

$$V_1(\pi, a, b) = U_1(c^y)\, w, \tag{26}$$

$$V_3(\pi, a, b) = \beta \int J_3(\pi', a', s(a' \mid a) + b)\, A_1(a' \mid a)\, da', \tag{27}$$

$$J_1(\pi', a', s(a' \mid a) + b) = \theta V_1(\pi', a', b'), \tag{28}$$

and

$$J_3(\pi', a', s(a' \mid a) + b) = U_1(c^{o'}). \tag{29}$$

The perfectly-pooled steady state. Now, in a perfectly-pooled steady state all young agents will consume the same amount, c^y.[11] Likewise, all old agents will have the identical level of consumption, c^o. From (25), (27), and (29) it then transpires that

$$r = 1/(\beta\theta). \tag{30}$$

If $\theta = 1$ then $r = 1/\beta$, the standard result for the neoclassical growth model. Alternatively, when the parent cares more about his own utility than his offspring's, or when $\theta < 1$, it happens that $r > 1/\beta$. Here parents place a higher weight on present consumption relative to the dynasty's future consumption. This dissuades savings and drives up the interest rate. In a perfectly-pooled equilibrium insurance will sell at its actuarially fair price

$$q(a' \mid a) = A_1(a' \mid a)/r. \tag{31}$$

From (22) this will imply that

$$U_1(c^y) = \beta r U_1(c^o).$$

Therefore, $c^y < c^o$ when $\theta < 1$.

[11] If the economy starts out in a perfectly-pooled equilibrium, then it will remain there forever. The question about how a perfectly-pooled equilibrium arose to begin with is ignored. A classic application of the perfect-pooling concept is Lucas's [15] study on international asset pricing—see his analysis for more detail on this notion.

By using (26), (28), and (30) in (23), and (24), it can be deduced that[12]

$$m, n \begin{cases} > 0, & \text{if } \ wn + m + \phi < w[H(a, m, n) - a]/r, \\ = 0, & \text{if } \ wn + m + \phi \geqslant w[H(a, m, n) - a]/r, \end{cases}$$

$$1 = H_2(a, m, n)\, w/r \qquad (\text{when } \ m > 0),$$

and

$$1 = H_3(a, m, n)/r \qquad (\text{when } n > 0).$$

These are the same conditions as (12), (13), and (14). Therefore, the efficient allocation can be supported by a competitive equilibrium with complete insurance markets. Markets are still efficient even when parents do not care about their offsprings as much as themselves.

4.1. *Numerical Example One*

An example of the efficient markets equilibrium will now be provided. Certain aspects of this example will be maintained in the subsequent two examples.[13] Additionally, some parameter values are chosen so that certain features of Example Two, which analyzes the economy with incomplete markets, are in accord with the U.S. data. Take the unit of time for a period to be 20 years.

[12] Deriving the threshold condition is a little less straightforward. Substituting equation (21) into (19) gives a young parent's lifetime budget constraint.

$$c^y + m + \phi I(\pi', a) + wn + \int q(a' \mid a)[c^{o'} + b'/r]\, da' = w\pi + \int q(a' \mid a)\, b\, da'.$$

So, all a young agent cares about is the present-value of his income, $w\pi + \int q(a' \mid a)\, b\, da'$, not how it is split up between wages and bequests. Hence, the young agent's value function can be rewritten as $V(\pi, a, b) = W(w\pi + \int q(a' \mid a)\, b\, da', a)$. In a perfectly-pooled steady state this further simplifies to $V(\pi, a, b) = W(w(\pi - a) + wa + b/r, a)$, since $q(a' \mid a) = A_1(a' \mid a)/r$. Now imagine solving problem (18) subject to the additional constraint that $m, n > 0$; i.e., that the agent's child becomes skilled. Let m and n denote the optimal solutions for money and time. Hence $\pi' = H(a, m, n) > a$. It costs $wn + m + \phi$ in terms of current resources to provide an individual's child with an extra $w[H(a, m, n) - a]$ units of labor income. Now, given the form of the value function, $w[H(a, m, n) - a]$ in labor income is worth the same to the child as $rw[H(a, m, n) - a]$ in bequests. But, as is evident from the lifetime budget constraint, leaving $b' = rw[H(a, m, n) - a]$ in bequests costs only $\int q(a' \mid a)\, w[H(a, m, n) - a]\, da' = w[H(a, m, n) - a]/r$ in terms of current resources. Therefore, in order to skill the child it must transpire that

$$wn + m + \phi < w[H(a, m, n) - a]/r.$$

[13] The examples presented are intended merely to illustrate the theory.

Tastes. Suppose that parents care about their children as much as they care about themselves; i.e., let $\theta = 1$. The discount factor is set so that $\beta = 0.91^{20} = 0.15$. From (30) this implies that in the efficient markets case, the (annualized) interest rate will be 9.9 percent. This value for the discount factor is selected so that the incomplete markets example can replicate the interest rate and investment-to-GDP ratio observed in the U.S. economy.

Production. Let production be given by a Cobb–Douglas production function so that

$$\mathbf{o} = O(\mathbf{k}, \mathbf{l}) = z\mathbf{k}^{\alpha}\mathbf{l}^{1-\alpha}.$$

In the U.S. economy labor's share of income is about 64 percent. So, set $\alpha = 0.36$. In the U.S. capital depreciates about 10 percent a year implying that $\delta = 1 - (1 - 0.10)^{20} = 0.88$.

Ability and productivity. Assume abilities lie in the discrete set $\mathscr{A} = \{a_1, a_2, ..., a_{15}\}$ and evolve in line according with a 15-state Markov chain. In particular, suppose that

$$A_{ij} = \Pr[a' = a_j \mid a = a_i].$$

The Markov chain for ability is tuned, following the procedure of Tauchen [23], to match the stochastic process $\ln a' = \iota(1-\omega) + \omega \ln a + \sigma\sqrt{1-\omega^2}\,\zeta$, where $\iota = 1.0/(1-0.35)$, $\omega = 0.35$, $\sigma = 0.45$ and $\zeta \sim N(0, 1)$. Next, little is known about the production function for human capital accumulation. Suppose that

$$H(a, m, n) = a^{\chi}[\tau n^{\varepsilon} + (1-\tau)\, m^{\varepsilon}]^{\rho/\varepsilon} + a, \qquad \varepsilon \leqslant 1. \tag{32}$$

For now simply assume that $\chi = 1.55$, $\tau = 0.65$, $\varepsilon = 0.32$, and $\rho = 0.16$. The fixed cost of becoming skilled is set so that $\phi = 0.13$.

At this stage simply take the choice of parameters values as given for the stochastic process governing ability and the human capital production function. They have been picked so that distribution of income arising in the incomplete markets model is in congruence with U.S. observation. This choice of parameter values is discussed in further detail in Example Two.

Algorithm. The equilibrium is computed as follows: To begin with note from (30) that, given a value for $1/(\beta\theta)$, the interest rate is known. Since the production function exhibits constant returns to scale this implies from (15) that $\mathbf{k}/\mathbf{l}$ is known too, since $O_1(\cdot)$ is homogeneous of degree zero. Consequently, the equilibrium wage rate $w = O_2(\cdot)$ is also known, since $O_2(\cdot)$ also depends solely on the $\mathbf{k}/\mathbf{l}$ ratio. Given w, equations (13) and (14) can then be used to compute $M(a)$ and $N(a)$ for each value of a. The solutions for w, $M(a)$ and $N(a)$ are then used to calculate the threshold level of ability, a^*, using (12). Last, for the equilibrium to be meaningful, the child-care market clearing condition (5) must hold.

4.1.1. *Results.* The upshot of the example is shown in Fig. 1, which plots the ability and productivity distributions for the population. These distributions are represented by step functions portraying the relevant histogram. The threshold level of ability lies at about the 6th decile; i.e., only the top 40 percent become skilled. There is a jump in the productivity distribution at this point. Also, observe that the productivity distribution is more skewed than the ability one. For future reference, let $\mathscr{W}$ denote the set of productivities that obtains in the efficient markets equilibrium. The fact that high-ability individuals have more time and resources invested in them amplifies wage inequality. This isn't an issue in an efficient markets equilibrium, since all actors enjoy the same level of consumption.

Impure altruism. Now consider the case where altruism is impure. Specifically, let $\theta = 0.5$. When parents care less about their children they

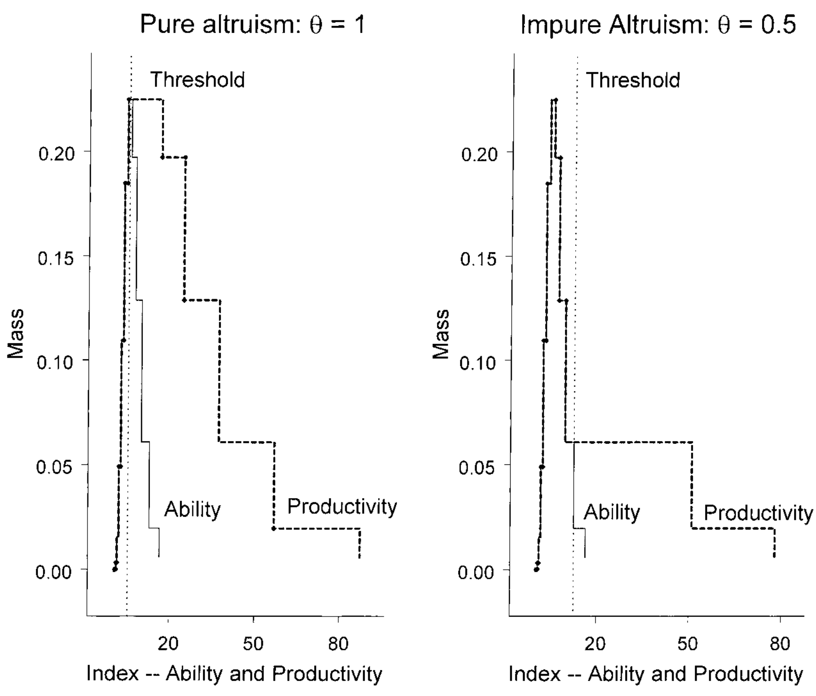

FIG. 1. Ability and productivity distributions—efficient markets case.

leave less in bequests. Hence, aggregate savings will be less and the steady-state interest rate higher. This fact can be seen immediately from (30). The (annualized) interest rate rises from 9.9 to 13.8 percent. The capital-labor ratio drops by about 110 percent. Additionally, one would expect that parents will now invest less in their children too. They do. The aggregate amounts of money and time invested in children fall by 278 percent and 251 percent, respectively. As a result, output drops by 106 percent. This translates into a decrease in consumption. When altruism is pure individuals consume an equal amount in each period, since the interest rate is equal to the rate of time preference. When altruism is impure their consumption profile slopes up over time, since the interest rate is higher than their discount factor. Consumption when young falls by a 120 percent, while consumption when old drops by 86 percent. While this equilibrium may seem horrifying relative to the previous one, remember that it is still efficient. Last, observe from Figure 1 that inequality is reduced.

The standard overlapping generations model. Consider the case where $\theta = 0$. Now, as $\theta \to 0$ equation (30) implies that $r \to \infty$. This isn't the standard overlapping generations model, however, as might appear at first glance. As the old care less about their offspring they borrow more against their children's income. This drives up the interest rate. In the standard overlapping generations model the old can't borrow against their offspring's income; that is, $\theta = 0$ and $b' \geqslant 0$. In this setting no parent would invest in his child. Hence, the steady-state supply of labor will be $\mathbf{l} = E[a]$. Next, each adult will save according to $\max_s\{U(wa - s/r) + \beta U(s)\}$. This yields the standard efficiency condition $U_1(wa - s/r) = \beta r U_1(s)$. Now, suppose that $U(c) = [c^{1-\mu} - 1]/(1-\mu)$. Then the solution for savings will be given by $s = S(a; w, r) = wa/[(\beta r)^{-1/\mu} + r^{-1}]$. The steady-state stock of capital is given by $\mathbf{k} = E[S(a; w, r)/r]$. This allows the wage and interest rates to be expressed as $w = O_2(E(s)/[rE(a)]) = (1-\alpha)[E(s)/(rE(a))]^{\alpha}$ and $r = O_1(E(s)/[rE(a)]) + (1-\delta) = \alpha[rE(a)/E(s)]^{1-\alpha} + (1-\delta)$. Finally, it is easy to deduce that $E[a]/E[s] = [(\beta r)^{-1/\mu} + r^{-1}]/w = \{[(\beta r)^{-1/\mu} + r^{-1}]/[(1-\alpha)\, r^{-\alpha}]\}^{1/(1-\alpha)}$. Therefore, $r = r[\alpha/(1-\alpha)][(\beta r)^{-1/\mu} + r^{-1}] + (1-\delta)$.

In the standard overlapping generations model the interest rate is 6.2 percent (when $\mu = 2.0$), below the 9.9 percent for the efficient markets equilibrium. The capital/labor ratio is higher by 110 percent. The capital stock is only slightly higher, though, about 23 percent. The reason is that the aggregate stock of labor is much smaller (87 percent or so), since there is no investment in children. This translates into aggregate consumption being 50 percent lower. The coefficient of variation in labor income is the same as the coefficient of variation in ability, or 0.45. Therefore, wage inequality is much lower in the standard overlapping generations model.

With the efficient markets equilibrium in mind, it is now possible to discuss various sources of inefficiencies in a decentralized system.

5. LACK OF INSURANCE AND LOAN MARKETS

The setting. The idealized world modeled above assumes that each parent can buy insurance on the ability of his grandchild. Those parents who draw a low-ability child are compensated with a cash payment financed by premiums paid by parents with a high-ability kid. Further, it also assumes that each parent can pass on a debt to his child. It's time to come down from this rarefied peak.

Suppose that parents can no longer buy or sell insurance. Instead they are free to trade one-period bonds subject to the proviso that they cannot pass on any debts to their offspring. Hence, they can self insure against the ability of their descendents by accumulating a stockpile of assets. Let b denote the (nonnegative) bequest a young adult inherits upon his parent's death and b' represent the amount that he will leave his child. The amount of savings that a young adult carries over for his old age will be given by s.

The non-negativity of bequests rules out a credit market. Adults with low productivity and high-ability children are unable to borrow in order to undertake the efficient amount of investment in their children. Public education might mitigate this inefficiency somewhat. For instance, if a child's ability is currently not known and is independently distributed across generations, or if the productivity of investment is independent of the child's ability level, then efficiency dictates a uniform level of investment in all children regardless of ability. Borrowing constrained adults may undertake lower investments.[14]

Choice problems. After the birth of his child, a young adult's state of the world will be given by his productivity, π, the ability of his offspring, a, and the bequest he will receive from his parent, b. At this stage, the only randomness in his life will be the ability level of his grandchild, a'. The dynamic programming problem facing a young parent is

$$V(\pi, a, b) = \max_{m, n, s \geqslant -b} \left\{ U(c^y) + \beta \int J(\pi', a', b+s)\, A_1(a' \mid a)\, da' \right\}, \quad (33)$$

[14] Borrowing constraints may be a factor in limiting college attendance, too. The situation here is different for two reasons: first, a young adult is presumably now deciding about his own educational inputs and, second, is borrowing against his own future income. That is, the young adult is issuing a claim against his own income and not against his descendents's incomes.

subject to (1) and

$$c^y + m + \phi I(\pi', a) + wn + s/r = w\pi.$$

Here

$$J(\pi', a', s+b) = \max_{b' \geqslant 0} \{U(c^{o'}) + V(\pi', a', b')\}, \tag{34}$$

subject to

$$c^{o'} + b'/r = s + b.$$

When the individual is old he will have a wealth level of $s+b$, a grown child with productivity π', and a grandchild of ability a'. At this time the agent will have to decide how much to leave to his adult child in bequests or b'. Problem (34) describes the decision making at this time. Therefore, $J(\cdot')$ is the indirect utility function for the old adult. Denote the decision rules for s, m, n, and b' that arise out of these problems by $s = S(\pi, a, b)$, $m = M(\pi, a, b)$, $n = N(\pi, a, b)$, and $b' = B(\pi', a', s+b)$.

If the young parent chooses not to educate his offspring then $\pi' = a$ and $m = n = 0$. If the individual chooses to educate his offspring then $\pi' > a$ and $m,\ n > 0$. The first-order necessary conditions for the young adult are[15]

$$s\colon U_1(c^y) = r\beta \int J_3(\pi', a', s+b)\, A_1(a' \mid a)\, da',$$

$$m\colon U_1(c^y) = \beta H_2(a, m, n) \int J_1(\pi', a', s+b)\, A_1(a' \mid a)\, da' \qquad \text{(when } m > 0\text{)},$$

and

$$n\colon U_1(c^y)\, w = \beta H_3(a, m, n) \int J_1(\pi', a', s+b)\, A_1(a' \mid a)\, da' \qquad \text{(when } n > 0\text{)}.$$

The last two equations imply that

$$wH_2(a, m, n) = H_3(a, m, n). \tag{35}$$

Equation (35) is also implied by (9) and (10). Therefore, while the lack of insurance might influence the level of investment in a child as measured by the attained level of productivity, π', it does not distort the decision about whether to invest cash, m, or time, n.

[15] Assume that $U_1(0) = \infty$ so that the individual will avoid hitting the borrowing constraint at all cost.

The steady state. Again focus on a stationary equilibrium for the economy. In a competitive equilibrium the interest and wage rates will once again be given by (15) and (16). In a stationary equilibrium the time-series mean of some variable for the agent will also equal the cross-sectional average across agents at any point in time. The aggregate supplies of capital and labor will be given by[16]

$$\mathbf{l} = E[\pi] - E[n],$$
$$\mathbf{k} = E[s/r + b/r].$$

5.1. *Numerical Example Two*

Setup. An example of the incomplete markets equilibrium will now be computed. At this point the momentary utility function needs to be parameterized, so let

$$U(c) = \frac{c^{1-\mu} - 1}{1-\mu}.$$

Let the coefficient of relative risk aversion assume a standard value of 2 so that $\mu = 2$. Retain the specification of tastes, technology, ability and productivity from the previous example. Hence, $\alpha = 0.36$, $\beta = 0.15$, $\theta = 1.0$, $\delta = 0.88$, $\iota = 1.0/(1-0.35)$, $\omega = 0.35$, $\sigma = 0.45$, $\chi = 1.55$, $\tau = 0.65$, $\varepsilon = 0.32$, $\rho = 0.16$, and $\phi = 0.13$.

ALGORITHM. Problems (33) and (34) are computed on a discrete space. Specifically, assume that $\pi \in \mathscr{P} \equiv \{\pi_1, ..., \pi_{100}\} \supset \mathscr{A} \cup \mathscr{W}$, $s + b \in \mathscr{S} \equiv \{v_1, ..., v_{125}\}$, and $b \in \mathscr{B} \equiv \{b_1, ..., b_{125}\}$.[17] Problem (33) can be rewritten as

$$V(\pi_i, a_j, b_k) = \max_{v \in \mathscr{S}, \pi' \in \mathscr{P}} \Big\{ U(w\pi + b_k/r - C(a_j, \pi'; w) - v/r) \quad (36)$$
$$+ \beta \sum_{l=1}^{15} J(\pi', a_l, v)\, A_{jl} \Big\},$$

[16] Let $D^y(\pi, a, b)$ represent the stationary distribution across young agents. Now, the distribution $A(a' \mid a)$ and the decision-rules $M(\pi, a, b)$, $N(\pi, a, b)$, $S(\pi, a, b)$, and $B(\pi', a', s + b)$ define a transition operator $T^y(\pi', a', b' \mid \pi, a, b)$. The stationary distribution D^y must solve $D^y(\pi', a', b') = \int T^y(\pi', a', b' \mid \pi, a, b)\, dD^y(\pi, a, b)$. Hence, $\mathbf{l} = \int [\pi - N(\pi, a, b)]\, dD^y(\pi, a, b)$. Last, the distribution over old agents, $D^o(\pi, a, s_{-1} + b_{-1})$, will be defined by $D^o(\pi', a', s+b) = \int T^o(\pi', a', s+b \mid \pi, a, b)\, dD^y(\pi, a, b)$, where the form of transition operator, T^o, will depend on A, M, N, and S. Therefore, $\mathbf{k} = [\int S(\pi, a, b)\, dD^y(\pi, a, b) + \int B(\pi', a', s+b)\, dD^o(\pi', a', s+b)]/r$.

[17] Recall that $\mathscr{W}$ is the set of productivities that emerges in the efficient markets equilibrium.

where

$$C(a, \pi'; w) = \begin{cases} \min_{m, n} \{m + \phi + wn : \pi' = H(a, m, n)\}, & \text{if} \quad \pi' > a, \\ 0, & \text{if} \quad \pi' = a. \end{cases} \tag{37}$$

Observe that equations (1) and (35) solve (37). Also, note that it is easy to recover the solution for s from the above problem since $s = \upsilon - b_k$. Likewise, problem (34) reads

$$J(\pi_i, a_j, v_k) = \max_{b' \in \mathscr{B}} \{U(\upsilon_k - b'/r) + V(\pi_i, a_j, b')\}. \tag{38}$$

Now, to compute the solution for J one needs to know the solution for V and vice versa. This is a fixed-point problem. This problem is solved using the following iterative scheme. Suppose that one enters some iteration j with a guess for V, denoted by V^j. Given the interest rate, r, and the guess, V^j, one can then solve (38) to obtain a guess for J, represented by J^j. Then a revised guess for V, or V^{j+1}, can be obtained by computing the solution to (36), given J^j, r and w. And so the algorithm goes on until $V^{j+1} \to V^j$ and $J^{j+1} \to J^j$. Of course one needs to compute the solutions for the equilibrium interest and wage rates, r and w. The details of the algorithm are in the appendix.

5.1.1. *Results.*

Precautionary savings. To begin with, the (annualized) interest rate in the incomplete markets economy is 5.0 percent. This is somewhat shy of the 6.9 percent return on capital reported by Cooley and Prescott [4]. It is also less than the (annualized percentage) rate of time preference of $(1-\beta^{1/20}) \times 100\% = 9.0$ percent. The investment-to-GDP ratio is 0.13, close to the 0.11 observed in the postwar U.S. As has been noted by Aiyagari [1] and Laitner [12], in economies with uninsured idiosyncratic risk individuals will tend to engage in precautionary saving. That is, they build up buffer stocks of financial assets to self insure against a run of bad luck. These precautionary savings drive down the interest rate. As a result of this precautionary savings, the capital stock in the incomplete markets economy is 147 percent higher than in the efficient markets case. In the model, $b/(b+s) = 1/3$; that is, one third of total wealth is made up by intergenerational transfers. Modigliani [16] reports that estimates of this number for the U.S. range from $1/5$ to $4/5$.

Additionally, individuals invest 60 percent more money in children, but about the same amount of time as before. Now, 75 percent of children become skilled as opposed to 41 percent previously. Therefore, borrowing constraints (in the presence of idiosyncratic risk) do not necessarily lead to underinvestment in children, as is typically presumed.[18] It does lead to misinvestment, however. The total supply of labor in market production is now 1.0 percent lower. This transpires because human capital investment is not directed toward the most able individuals.

To see the effect that idiosyncratic risk has on precautionary savings, cut the standard deviation of the ability shock by half so that $\sigma = 0.22$. The mean level of ability remains unchanged. The interest rate rises from 5.0 to 6.2 percent, while the capital stock drops by 61 percent. Both the money and time invested in children falls (7.5 percent and 39.8). The number of children who become skilled also decreases by 3.5 percentage points.

Inequality. Figure 2 plots the distributions of ability and productivity. The ability distribution is portrayed by a step function while the productivity distribution is illustrated by a discrete density function. The distribution of productivities is approximately lognormal and resembles the U.S. earnings distribution—as documented by Knowles [11]. The coefficient of variation in productivity is about 0.78, close to the 0.77 observed in the data. Likewise, the Gini coefficient for the distribution of income in the model is 0.39 versus 0.35 in data. Solon [20] reports that for the U.S. the correlation of earnings across generations is about 0.52; in the model it is 0.64.[19] The distribution of productivities does not arise in a straightforward manner from the distribution of abilities. The distribution of productivities is more skewed than the distribution of abilities, as can be seen from Fig. 2. The match between the model and the U.S. data is obtained by picking the parameters governing the ability distribution in conjunction with the parameters governing the production of human capital.

Does the presence of incomplete insurance increase income inequality? The answer is no. There is less inequality in productivity across individuals in the incomplete markets world relative to the efficient one. This is readily seen by comparing Figs. 1 and 2. The Gini coefficient in the efficient markets case is 0.51, as opposed to 0.39 here. The ratio of productivities earned by the top 5 percent relative to the bottom 5 percent is 20.57, compared with 27.15 for the efficient markets world. In the efficient markets world inequality isn't a problem; however, since everybody enjoys the same consumption due to perfect risk sharing. There may be reasons

[18] This seems to derive from the higher level of physical wealth in economy. Hence, parents can invest more cash in their kids. Additionally, as the interest rate falls parents substitute out of physical capital and into human capital.

[19] For a review of this literature, see Stokey [21]. The assumed degree of persistence in ability ($\omega = 0.35$) is not high. According to Hernnstein and Murray [9] the intergenerational correlation in AFQT scores lies somewhere between 0.4 and 0.8. Hence, in the model, about one half of the persistence in income comes from market structure.

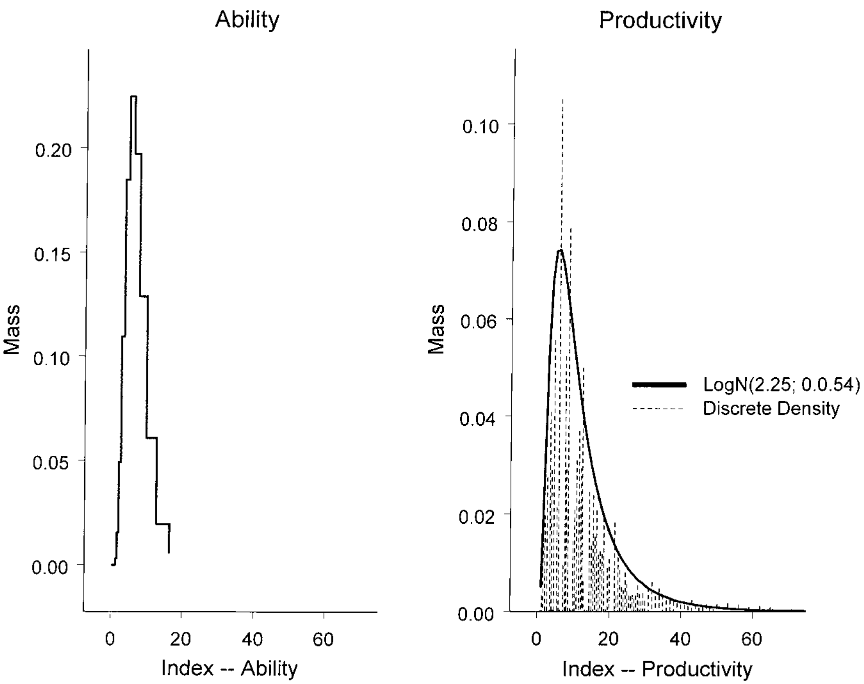

FIG. 2. Ability and productivity—incomplete markets case.

why inequality may be less in the incomplete markets world. First, borrowing constraints may reduce the ability of parents to invest in highly talented children, arguing for lower dispersion. Second, given the lack of insurance markets, parents may want to invest more in their children's human capital (irregardless of ability) to insure against idiosyncratic risk—recall that the interest rate is lower in this world.

Welfare gain from completing markets. So, what is the welfare loss that arises from the uninsured idiosyncratic risk? Some care must be exercised when assessing this. Steady-state output is 52 percent higher in the incomplete markets economy, as compared with the efficient one. Average consumption is 43 percent higher too. Utility is higher as a consequence. Surely, the average agent can't be better off in the incomplete markets economy as opposed to the efficient one. The answer to this apparent contradiction lies in the comparison of steady states. Recall that in the incomplete markets economy there is overaccumulation due to precautionary savings. This leads to high levels of output, average consumption, and utility.

Now imagine starting the efficient markets economy from the steady-state capital stock and productivity distribution that obtain in the incomplete markets economy.[20] Over time this economy will converge to the efficient markets steady state. Would a young agent prefer the utility realized in this economy or the average level of expected utility level that obtains in the incomplete markets economy? Let $\{c_t^y, c_t^o\}_{t=0}^{\infty}$ be the path of consumptions that will arise in the efficient markets economy and $E[V]$ denote the average level of expected utility in the incomplete markets economy. The agent would be willing to increase his consumption in each period by $\lambda \times 100\%$ and still be happy to live in the efficient markets economy, where

$$\lambda = \left\{ \frac{E[V] + [1+\beta]/[(1-\mu)(1-\beta\theta)]}{\sum_{t=0}^{\infty} (\beta\theta)^t \, [(c_t^y)^{1-\mu} + \beta(c_{t+1}^o)^{1-\mu}]/(1-\mu)} \right\}^{1/(1-\mu)} - 1.$$

Observe that as the level of expected utility in the incomplete markets economy, $E[V]$, increases the fraction of efficient markets consumption that the agent would be willing to give up, or λ, falls. Clearly solving for λ requires computing the transitional dynamics for the efficient markets economy. The algorithm used to do this is detailed in the appendix.

It turns out that $\lambda = -0.63$, so that an individual would prefer to live in the efficient markets economy. Along the transition path from the incomplete to complete markets economy the individual temporarily increases his consumption as the economy runs down its stocks of physical and human capital. The time path for aggregate consumption is shown in Fig. 3, which also plots the evolution of the economy's productivity distribution.[21] The rapid convergence to the efficient-markets steady state should be expected given that a period is 20 years.

In fact more can be said than this. It is possible to compute the compensating variation for a person starting off from *any* initial condition, or (π, a, b)-combination, in the incomplete markets economy. Intuitively, one would expect that an agent with high values for π, a, and b would gain less from such a move than an individual with low values for these variables—recall that in the efficient markets economy all people within a given generation enjoy the same level of consumption. The distribution of

[20] As before, assume a perfectly-pooled equilibrium. Hence, within each generation all actors are equally well off.

[21] The left panel shows how the productivity distribution evolves over time. The initial distribution is portrayed by the discrete density function shown by the – – lines. The – · · · – line shows, in step-function form, the productivity distribution that obtains after one period. The solid line gives the final productivity, again in step-function form.

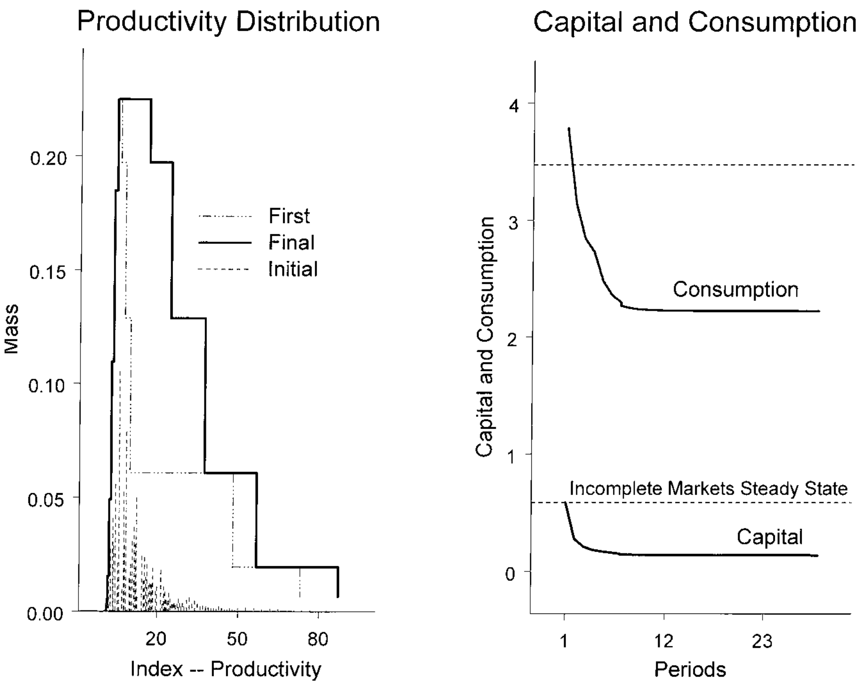

FIG. 3. Transitional dynamics—from incomplete to complete.

these compensating variations is plotted in Fig. 4. Note that *everybody* is made better off from the regime switch, although the person with lowest expected utility in the incomplete markets economy gains about 3 times as much as the person with the highest utility.

Impure altruism, again. Once again set $\theta = 0.5$, implying that parents care less about their children than themselves. How does the new equilibrium compare with the incomplete markets economy with pure altruism? The amount of time that parents invest in their childrens' human capital falls by 230 percent, while the amount of goods falls by 183 percent. They also leave 222 percent less in bequests. The fact that parents are investing less in the future leads to a rise in the equilibrium interest rate from 5.0 to 6.0 percent as the aggregate capital stock drops by 97 percent. The cut in human capital investment leads to 66 percent less efficiency units of labor being used in production. The net result of all of this is that output declines by 77 percent. As $\theta \to 0$ the model converges to the standard overlapping generations structure discussed in the previous section.

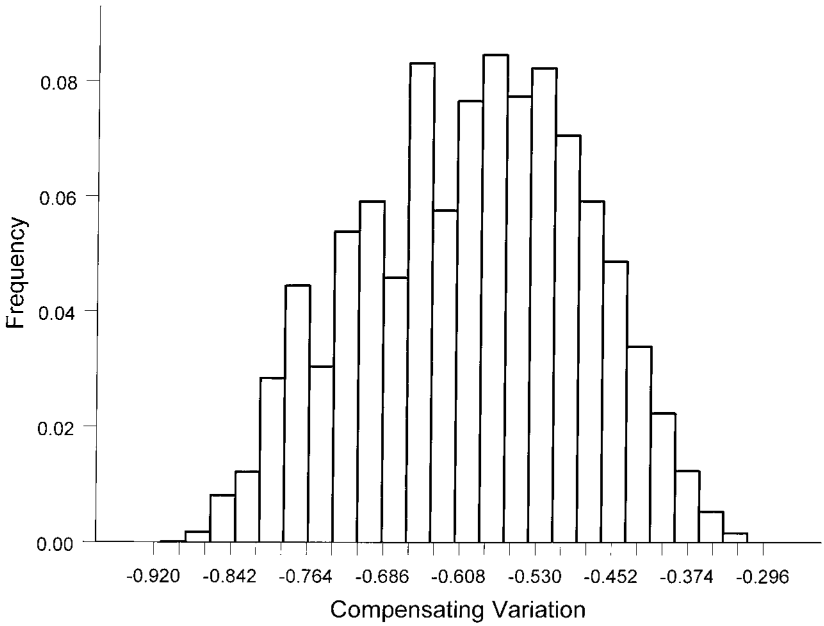

FIG. 4. Distribution of compensating variations.

6. LACK OF CHILD-CARE FACILITIES

The setting. The efficient equilibrium presumes that an efficient child-care market exists. Suppose not. Then, each parent must invest his own time in his child. Consider a parent of productivity π with bequest b who has a child of ability a. Assume that a parent of productivity π has a productivity $P(\pi)$ in nurturing his *own* child. For instance, on the one hand, $\pi = P(\pi)$ represents a "quality-time" world where a parent's productivity in child-care is the same as in the market. On the other hand,

$$P(\pi) = \begin{cases} \pi, & \pi \leqslant p^*, \\ p^*, & \text{otherwise}, \end{cases} \tag{39}$$

could be thought of as a world where child-care is a (relatively) low-productivity occupation that high-productivity agents have no real advantage at. Now, for each young parent it must transpire that

$$n/P(\pi) + l = 1.$$

In other words, for a parent of productivity π it costs $n/P(\pi)$ units of time to provide n efficiency units of child-care. A non-existent (or badly functioning) labor market in child-care will force highly productive adults to devote time to child-care instead of production.

Choice problems. The dynamic programming problem facing a young parent is

$$V(\pi, a, b) = \max_{s \geqslant -b, m, n} \left\{ U(c^y) + \beta \int J(\pi', a', b+s) \, A_1(a' \mid a) \, da' \right\},$$

subject to (1) and

$$c^y + m + \phi I(\pi', a) + s/r = w\pi(1 - n/P(\pi)).$$

Once again $J(\cdot')$ is defined by (34).

The first-order necessary conditions for the young adult are

$$s\colon U_1(c^y) = r\beta \int J_3(\pi', a', b+s) \, dA_1(a' \mid a),$$

and

$$m\colon U_1(c^y) = \beta H_2(a, m, n) \int J_1(\pi', a', b+s) \, dA_1(a' \mid a) \quad (\text{when } m > 0),$$

$$n\colon U_1(c^y) w\pi/P(\pi) = \beta H_3(a, m, n) \int J_1(\pi', a', b+s) \, dA_1(a' \mid a) \quad (\text{when } n > 0).$$

The last two equations imply that

$$[w\pi/P(\pi)] \; H_2(a, m, n) = H_3(a, m, n). \tag{40}$$

Equation (40) is similar to (35), with one exception. Now, the parent's relative productivity level in nurturing, $\pi/P(\pi)$, affects the decision about how much time to invest in child-care. The more productive the young parent is in the market vis à vis at home, the more he will favor investing money as opposed to time in his child, other things equal.

Before proceeding, note that the quality-time case is just simply uninteresting. If an individual is equally productive in child-care as market work then he would be indifferent between using his own time in child-care or using it at work. Consider a person of productivity π. To buy π units of quality time in child-care on the market (if it was available) would cost w units of consumption. The agent could supply the same amount of quality time himself and lose w in wage income. Hence, the lack of a child-care market would be inconsequential. Each parent could easily raise his own child and there would be no cost advantage in letting someone else do it. In the quality-time world the absence of a child-care market will *not* matter.

6.1. *Numerical Example Three*

Setup. The case where child-care is a (relatively) low-productivity occupation is now considered. The parameterization from the incomplete markets case (with pure altruism) will be retained. The same numerical algorithm used to solve the incomplete markets case is employed here. All that remains to be specified is the threshold level of productivity, p^*, in (39). It is assumed that this threshold lies at about the 50th percentile in productivity, implying that $p^* = 8.5$.

6.1.1. *Results.* Consumption and output both fall by about 6 percent, relative to the incomplete markets case with child-care. This is caused by a 78 percent drop in child-care time. The amount of goods invested in children only decreases by 7 percent, though. The fraction of children receiving no investment rises slightly from 25 to 28 percent. Now, the drop in consumption and output may seem small. This transpires for three reasons. First, the human capital production function (32) is very concave. Second, note that the welfare loss from an inefficient child-care market arises because high-productivity individuals must spend their time inefficiently at home raising their kids as opposed to working. There will be no loss for those agents with $\pi \leqslant p^*$. For an individual with productivity $\pi > p^*$ the loss will be $w(\pi/p^* - 1)$ per unit of child-care time. So, a large drop in consumption and output will require that $\pi - p^*$ is large and positive for a significant fraction of the population. This seems unlikely given the shape of the income distribution and the average earnings of child-care specialists—Fig. 5 portrays the situation using data generated from the model.[22] Here the jagged solid line shows the distortion, $(\pi/p^* - 1)$,

[22] One could argue that the market sector is more efficient at providing child-care than the home sector, say due to economies of scale or specialization. Suppose that the market sector is twice as efficient at looking after children relative to the home sector. To capture this, let $P(\pi) = \pi/2$, for $\pi \leqslant p^*$, and $P(\pi) = p^*/2$, otherwise. Now, there is a 164 percent drop in child-care time, while the amount of goods invested falls by 15 percent, relative to the incomplete markets case with child-care. Consumption and output are both reduced by 13 percent. Of course, one could just as easily argue that the market sector is less efficient at providing child-care than the home sector, due to incentive and other problems. For instance, a daycare provider may not care about your children as much as you do. A recent study financed by the National Institute on Child Health and Human Development found (as reported by the *The New York Times*, April 19th 2001) that children raised in daycare are three times as likely to experience behavioral problems as those raised primarily by their mothers. The study followed 1,100 children in 10 cities from a variety of child-care settings.

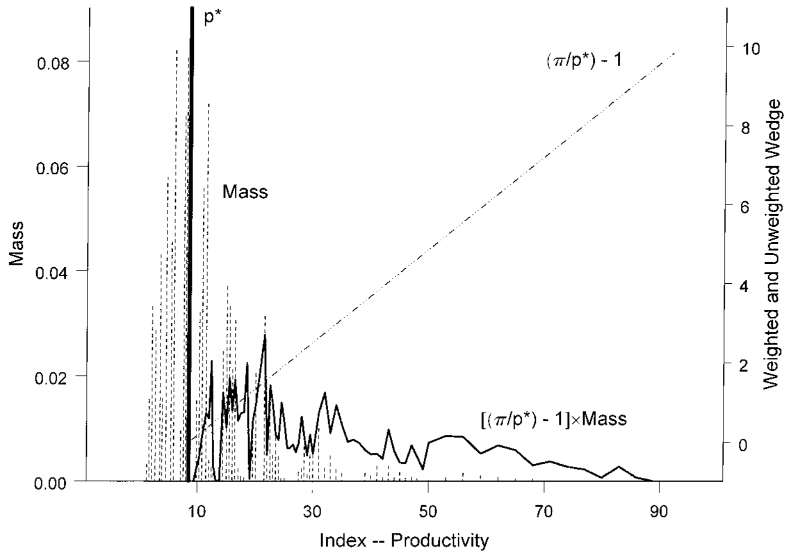

FIG. 5. Lack of child-care market—distortion.

weighted by the number of affected agents. Third, not much parental time is involved in the human capital development of children.[23]

7. CONCLUSIONS

When discussing the impact of imperfect financial markets, Arthur Okun [18, pp. 80–81] once said that "the most important consequence is the inadequate development of the human resources of the children of poor

[23] In the U.S., an average mother spends about 3.0 hours a week per child on direct child-care, according to Hill and Stafford [10, Table 17.6]. She had about 2.5 kids (in the postwar period). Direct child-care is defined to be activities such as "helping/teaching, reading/talking (including 'yelling at'), indoor playing, outdoor playing, medical care and other regular child-care such as feeding, dressing, supervising, and other direct interaction" (p. 427). The time spent drops off dramatically as a child ages. For instance, a high-school educated mother spends about 6.0 hours a week on these activities for a preschooler, but only about 1.7 hours a week for a child in school. Now, a parent only raises a child for about 18 of the 40 or so years that he works. And, the average household puts in about 54 hours of market per week. Thus, about $[2.5 \times (18/40) \times 3.0]/(2.5 \times (18/40) \times 3.0 + 54) \times 100\% = 5.9$ percent of a parent's time is spent on child-care, or about 2.4 percent per child. These numbers seem low. The estimates exclude any purchased time on direct child-care. In the incomplete markets model with child-care markets about $E[n]/E[a] \times 100\% = 2.7$ percent of the total feasible time available for child-care is used. In the framework without child-care markets only about 0.9 percent of available parental time is used. Any serious quantitative analysis would have to obtain accurate statistics on the amount of privately controlled inputs going into a kid's human capital production.

families—which, I would judge, is one of the most serious inefficiencies of the American economy today." A general equilibrium model was developed here where children differ by ability. Parents could invest time and goods in the development of their children's human capital. The model can be used to examine this type of claim. In a world with perfect financial markets parental investment in a child would be a function solely of the kid's ability. Financial markets aren't complete, however, in the real world. First, ideally an individual would like to insure against his grandchild's ability, as long as there is some randomness in it. Second, a parent cannot borrow against his child's future income in order to educate him today. Given this, the analysis is not as straightforward as Okun [18] and others presume. In fact in the numerical example presented, the absence of insurance markets and the presence of borrowing constraints did *not* lead to underinvestment in children—more money was invested in kids. The investment was inefficient, however, in that it was not directed toward the children with the highest ability.

Another market failure may be the lack of child-care markets. This too is more problematic than is typically believed. For this market failure to be severe, the returns in terms of a child's productivity to an extra unit of investment in time cannot fall off too dramatically with the level of investment. Additionally, there must be a significant number of parents whose productivity at work is greater than the productivity of the child-care specialist who will look after their child. This seems unlikely to be case. As such, it is likely to be rich people (doctors, lawyers, etc.) and not poor ones (janitors, restaurant waitresses, etc.) that will benefit the most from completing child-care markets.

Perhaps the problem of underinvestment in children is that altruism is impure: that is, parents do not care about their children as much as they care about themselves. Parents invest much less in their children when altruism is impure. Impure altruism, however, can't be labelled a market failure in the traditional sense. The equilibrium may still be Pareto optimal. Over time the lot of children in society has improved; they no longer work and they go to school. When analyzing this process, economists often tend to take agents' preferences as constant and model it as the outcome of technological progress. Historians and sociologists often view this process as arising from shifts in societal attitudes toward children, or changes in preferences. They arrive at this conclusion by analyzing changes in attitudes towards children and shifts in childrearing practices, etc.—see Stone [22, chp. 9]. Undoubtedly both technological and cultural forces are at play in determining the well-being of children. There is little an economist can say about how goods (here children) *should* (as opposed to do) factor into a person's tastes. This is a moral question that society may have to take a stand on.

APPENDIX: ALGORITHMS

Incomplete markets steady state. The algorithm used to compute the solution for the incomplete markets case will now be described. The other cases are computed in a similar manner.

Computing the competitive equilibrium for the incomplete market economy involves the following steps. To begin with, draw a random time series of T observations for a using the distribution function $\mathbf{A}$. Call this sample path $\{a_t\}_{t=0}^{T}$.

(1) Enter iteration j with a guess for the interest and wage rates, r and w, denoted by r^j and w^j.

(2) Given this guess, solve the choice problems (33) and (34).

(3) Simulate the decision rules for (33) and (34) T times using the randomly generated sample for the a's. To do this, start at the point (π_0, a_0, b_0). Use the decision rules from problem (33) to get s_0, π_1. Next, use the decision rule from problem (34) at the point (π_1, a_1, s_0+b_0) to obtain b_1. The decision rules for (33) can now be evaluated at the point (π_1, a_1, b_1) to get s_1, π_2. Proceed down the rest of the sample path in similar manner. Collect data on s, b', n, and π; that is the sequences $\{s_t\}_{t=1}^{T}$, $\{b_{t+1}\}_{t=0}^{T}$, $\{n_t\}_{t=1}^{T}$, and $\{\pi_t\}_{t=1}^{T}$. Calculate $E[s+b']$, $E[\pi]$, and $E[n]$, or the sample means for $s+b'$, π, and n.

(4) Compute a revised guess for the interest and wage rates, r^{j+1} and w^{j+1}. Since the focus is on a stationary competitive equilibrium, a natural way to do this would be to set

$$r^{j+1} = O_1(\mathbf{k}^j, \mathbf{l}^j) - \delta,$$

and

$$w^{j+1} = O_2(\mathbf{k}^j, \mathbf{l}^j) .$$

Now, in equilibrium aggregate savings will be given by $\mathbf{k}^j = E[s+b']/r^j$ and $\mathbf{l}^j = E[\pi] - E[n]$.

(5) Check if $metric(r^{j+1}, r^j)$ and $metric(w^{j+1}, w^j)$ fall below some specified tolerance. If so, stop. If not, go back to step 1.

The child-care market must clear for an equilibrium to prevail. This necessiates checking that the following condition holds: $\sum_{t=1}^{T} n_t \leqslant \sum_{t=1}^{T} a_{t-1} [1 - I(\pi_t, a_{t-1})]$, where again $I(\pi_t, a_{t-1}) = 1$ if $\pi_t > a_{t-1}$ and $I(\pi_t, a_{t-1}) = 0$ if $\pi_t = a_{t-1}$.

Complete markets transitional dynamics. Let the initial aggregate stock of capital be represented by $\mathbf{k}_0$ and the initial distribution of productivities be denoted by Π_0. Recall that these state variables arise from the incomplete-markets-economy steady state. The goal is to compute the economy's transition path to the efficient markets steady state. Pick a T, suitably large enough, so that convergence to the new steady state takes place within $T+1$ periods. Therefore, let $\mathbf{k}_t$, $\mathbf{l}_t$, $\mathbf{m}_t$ assume their steady-state values for all $t \geqslant T+1$. The algorithm works as follows:

(1) Enter iteration j with a guess for the time paths $\{\mathbf{k}_t\}_{t=1}^T$, $\{\mathbf{l}_t\}_{t=1}^T$, and $\{\mathbf{m}_t\}_{t=1}^T$ denoted by $\{\mathbf{k}_t^j\}_{t=1}^T$, $\{\mathbf{l}_t^j\}_{t=1}^T$, and $\{\mathbf{m}_t^j\}_{t=1}^T$. This implies a guess for $\{w_t\}_{t=1}^T$, denoted by $\{w_t^j\}_{t=1}^T$. Note that $\mathbf{k}_0$ and $E_0[\pi]$ are tied down by the initial condition.

(2) Start off at period 0. Now, given w_1^j, $\mathbf{m}_1^j$, and $\mathbf{k}_2^j$ solve for a_0^*, $M_0(a)$, $N_0(a)$, $\mathbf{l}_0$, and $\mathbf{k}_1$ using

$$M_0(a_0^*)+\phi+N_0(a_0^*)\,\overbrace{O_2(\mathbf{k}_0,\mathbf{l}_0)}^{w_0}$$
$$= w_1^j[H(a_0^*, M_0(a_0^*), N_0(a_0^*))-a_0^*]/\underbrace{[O_1(\mathbf{k}_1,\mathbf{l}_1^j)+(1-\delta)]}_{r_0}, \quad (41)$$

$$w_1^j H_2(a_0^*, M_0(a_0^*), N_0(a_0^*))$$
$$= \underbrace{[O_1(\mathbf{k}_1,\mathbf{l}_1^j)+(1-\delta)]}_{r_0},$$

$$w_1^j H_3(a_0^*, M_0(a_0^*), N_0(a_0^*))$$
$$= \underbrace{[O_1(\mathbf{k}_1,\mathbf{l}_1^j)+(1-\delta)]}_{r_0}\,O_2(\mathbf{k}_0,\mathbf{l}_0), \quad (42)$$

and

$$[O(\mathbf{k}_0,\mathbf{l}_0)+(1-\delta)\,\mathbf{k}_0-\mathbf{m}_0-\mathbf{k}_1]^{-\mu}$$
$$= \beta\theta[O_1(\mathbf{k}_1,\mathbf{l}_1^j)+(1-\delta)][O(\mathbf{k}_1,\mathbf{l}_1^j)+(1-\delta)\,\mathbf{k}_1-\mathbf{m}_1^j-\mathbf{k}_2^j]^{-\mu}. \quad (43)$$

Equations (41) to (42) derive from (8) to (10). Equation (43) is the Euler equation governing capital accumulation and is a rewritten version of (11). In any perfectly-pooled equilibrium, a young parent's Euler equation implies that $c_t^y = (\beta r)^{-1/\mu}\, c_{t+1}^o$. Additionally, it can be shown that for each dynasty $c_t^o = \theta^{-1/\mu}\, c_t^y$. Aggregating over agents, while using these two facts, gives $\mathbf{c}_t = (\beta\theta r)^{-1/\mu}\,\mathbf{c}_{t+1}$. This forms the basis for (43).

(a) Solving the above system of equations requires an inner loop. That is, given a guess for $\mathbf{l}_0$ and $\mathbf{k}_1$, first solve for a_0^*, $M_0(a)$, $N_0(a)$ using the first three equations. Then, revise the guess for $\mathbf{l}_0$ and $\mathbf{k}_1$ using the $\mathbf{l}_0 = E_0[\pi] - \int_{a_0^*} N_0(a)\, dA(a)$ and (43). Iterate until convergence in the answers for a_0^*, $M_0(a)$, $N_0(a)$, $\mathbf{l}_0$, and $\mathbf{k}_1$ is achieved. Exit the inner loop.

(3) Given this solution enter period 1 with the initial condition $\mathbf{k}_1$ and Π_1. Given w_2^j, $\mathbf{m}_2^j$, and $\mathbf{k}_3^j$ solve for a_1^*, $M_1(a)$, $N_1(a)$, $\mathbf{l}_1$, and $\mathbf{k}_2$ using the updated version of (41) to (43). Travel down the path in this fashion to get $\{\mathbf{k}_t\}_{t=1}^T$, $\{\mathbf{l}_t\}_{t=1}^T$, and $\{\mathbf{m}_t\}_{t=1}^T$. Use this solution for the revised guess $\{\mathbf{k}_t^{j+1}\}_{t=1}^T$, $\{\mathbf{l}_t^{j+1}\}_{t=1}^T$, and $\{\mathbf{m}_t^{j+1}\}_{t=1}^T$.

(4) Repeat until convergence in $\{\mathbf{k}_t^j\}_{t=1}^T$, $\{\mathbf{l}_t^j\}_{t=1}^T$, and $\{\mathbf{m}_t^j\}_{t=1}^T$ is obtained. Exit the algorithm. Additionally, for the solution to be meaningful, it must also be checked that the child-care market-clearing condition (5) always holds along the equilibrium path.

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